Lecture 5

Red-Black Trees, Height Bound, Insertion

Source: Introduction to Algorithms, CLRS

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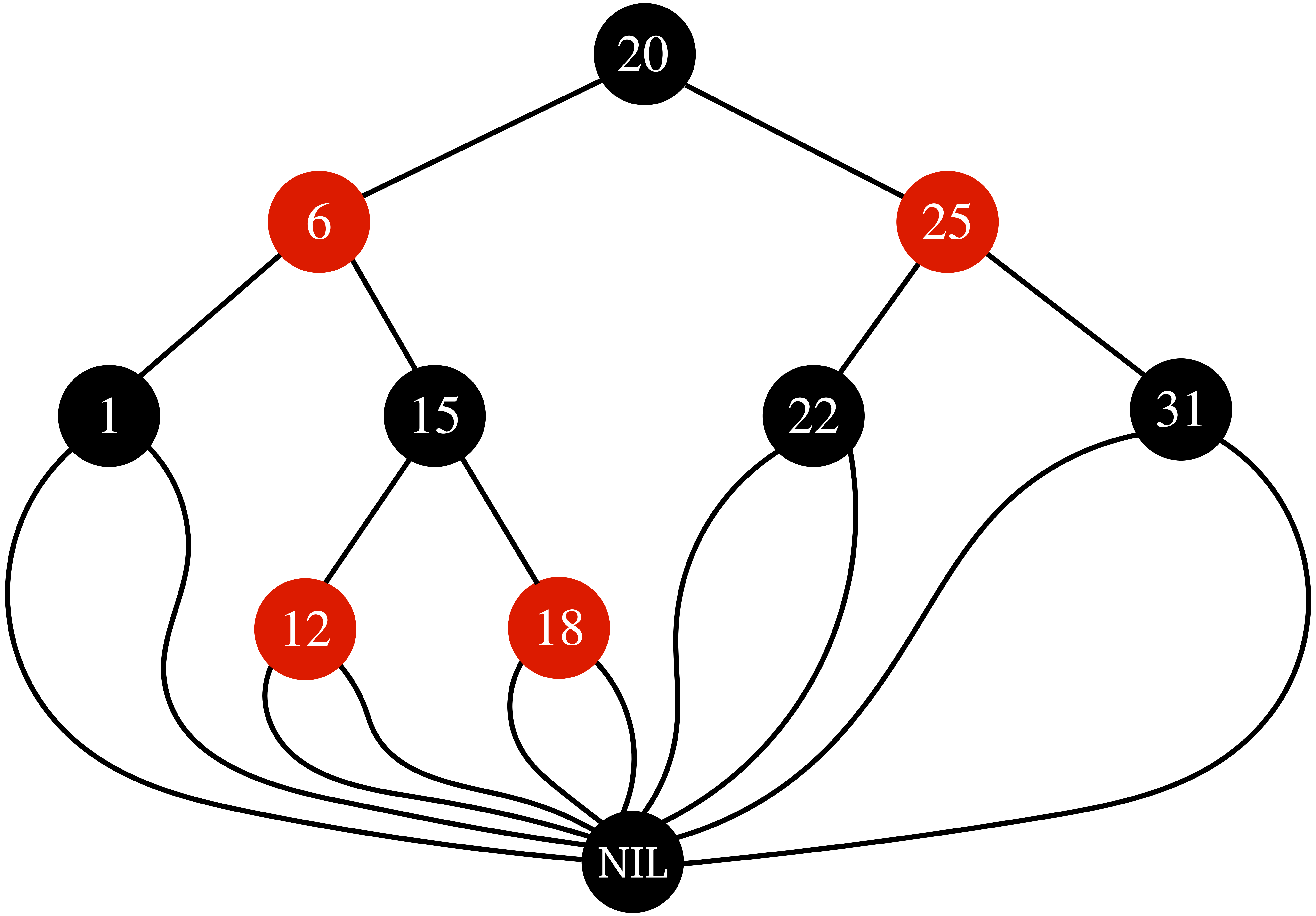

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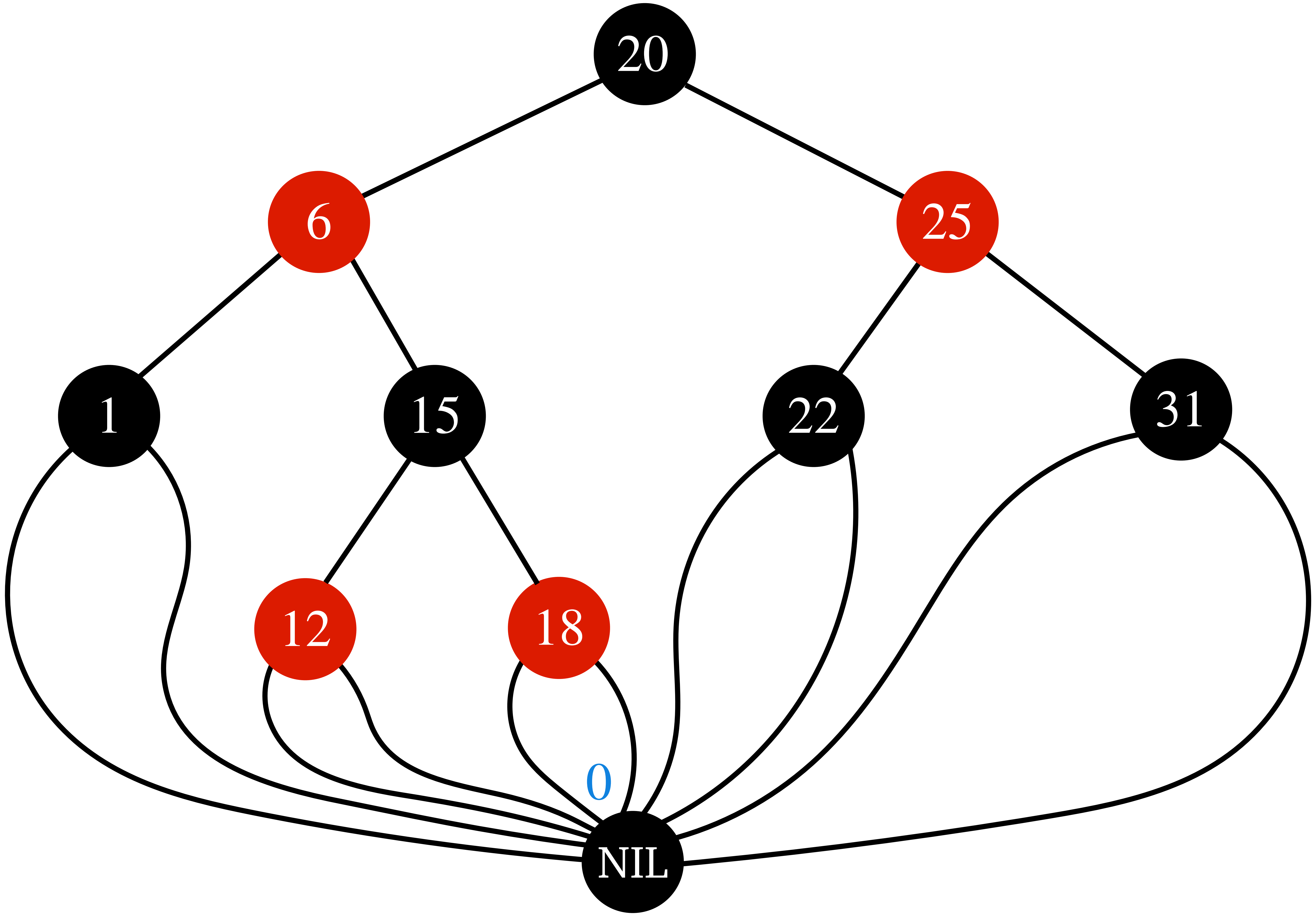

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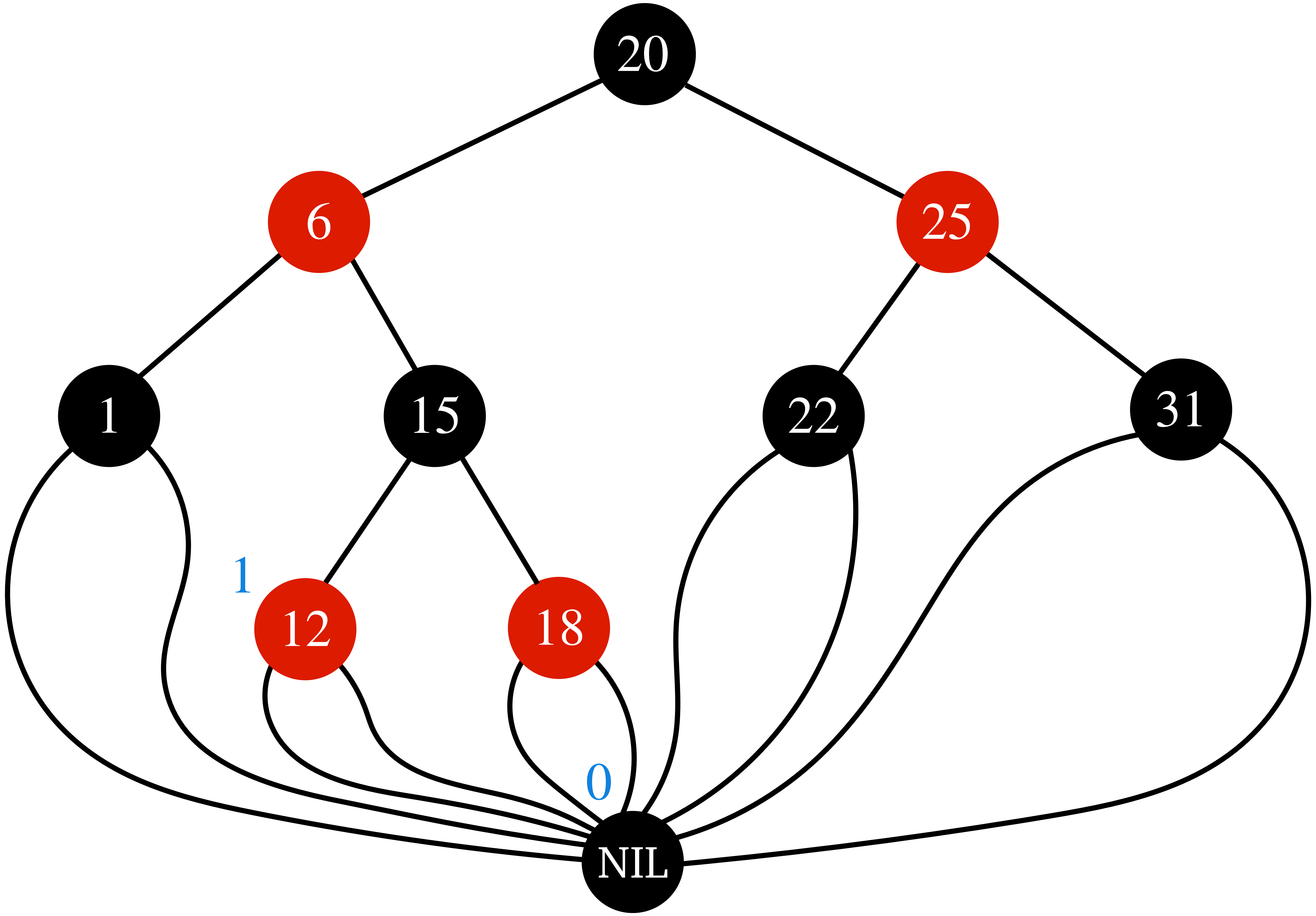

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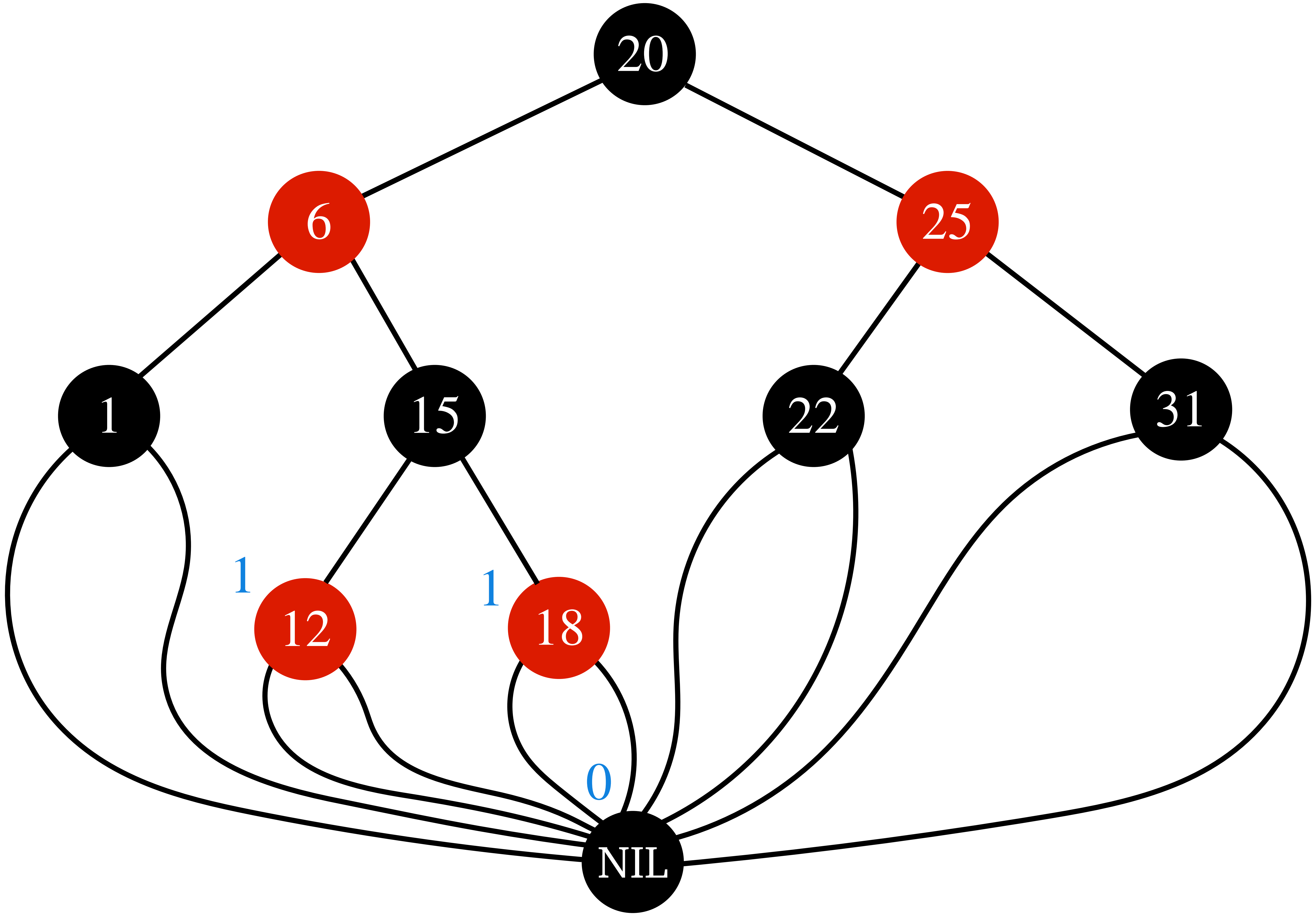

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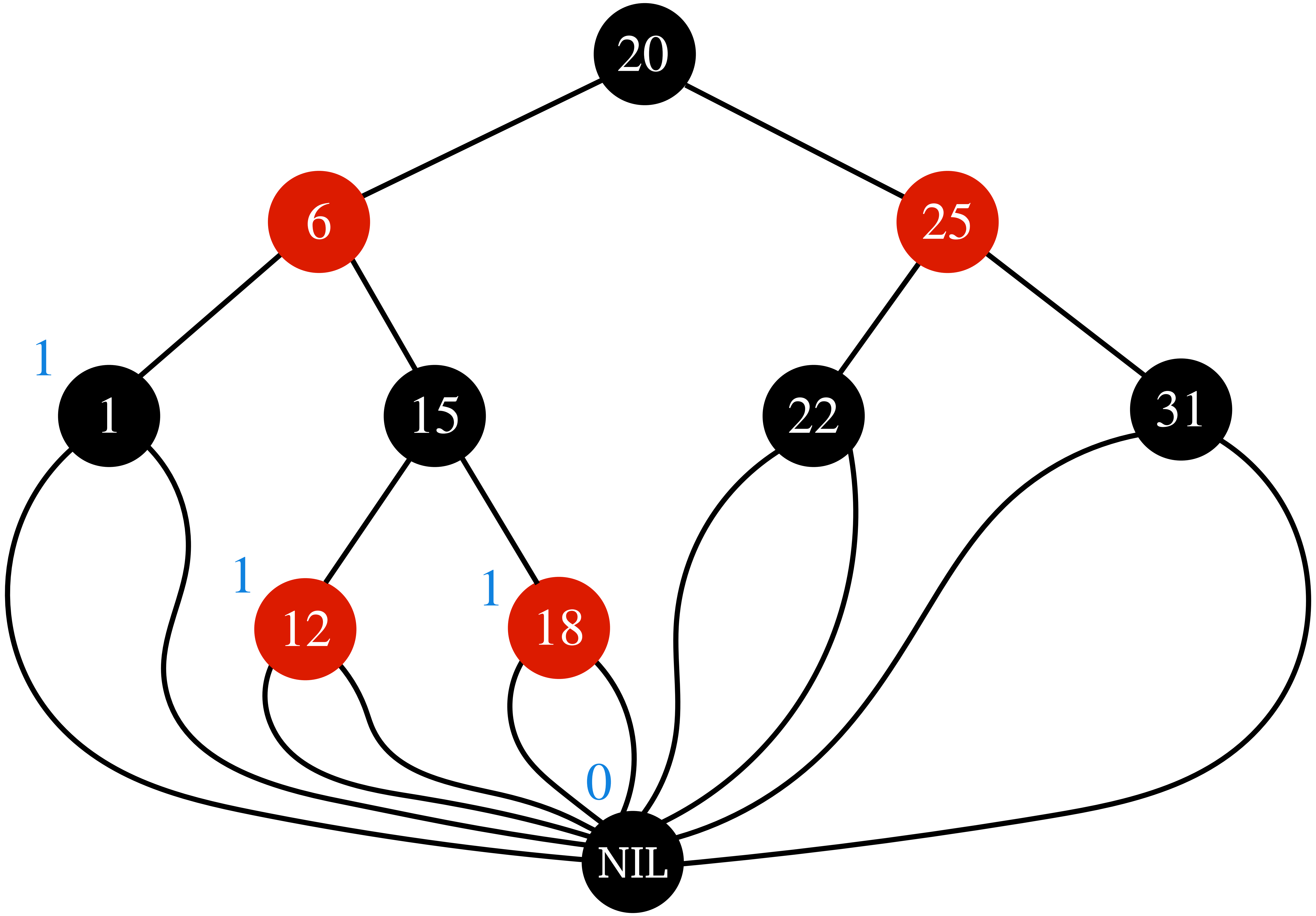

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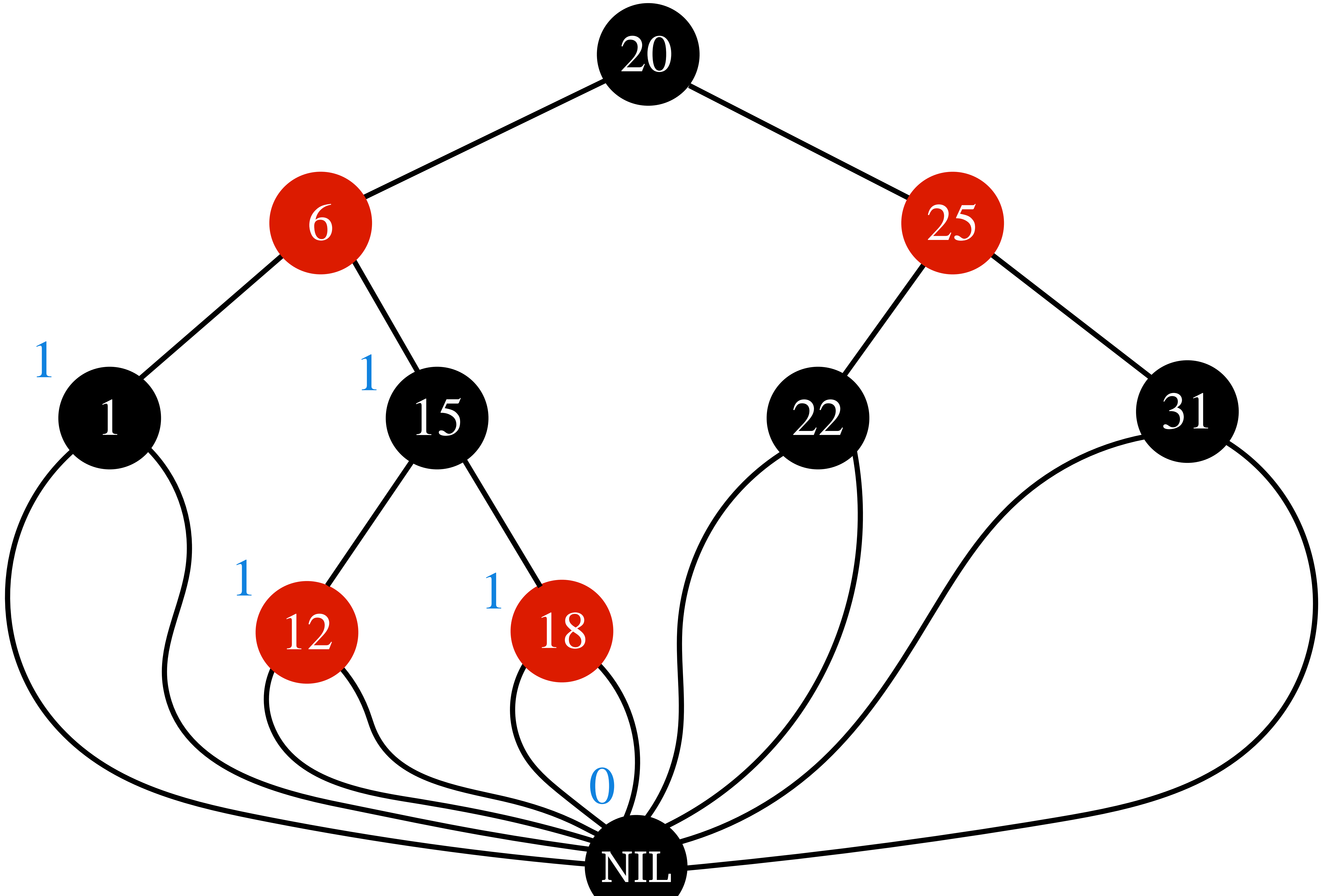

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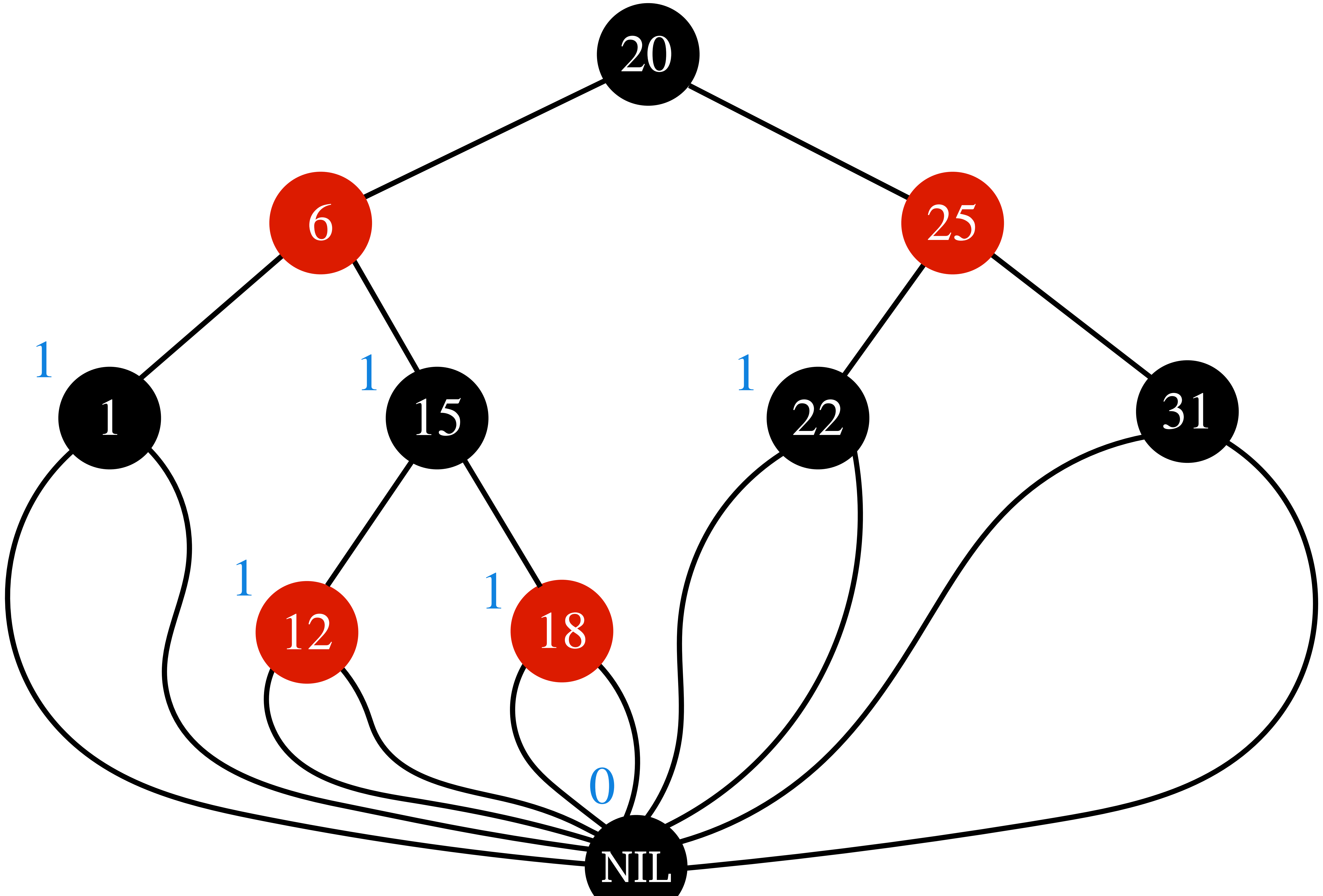

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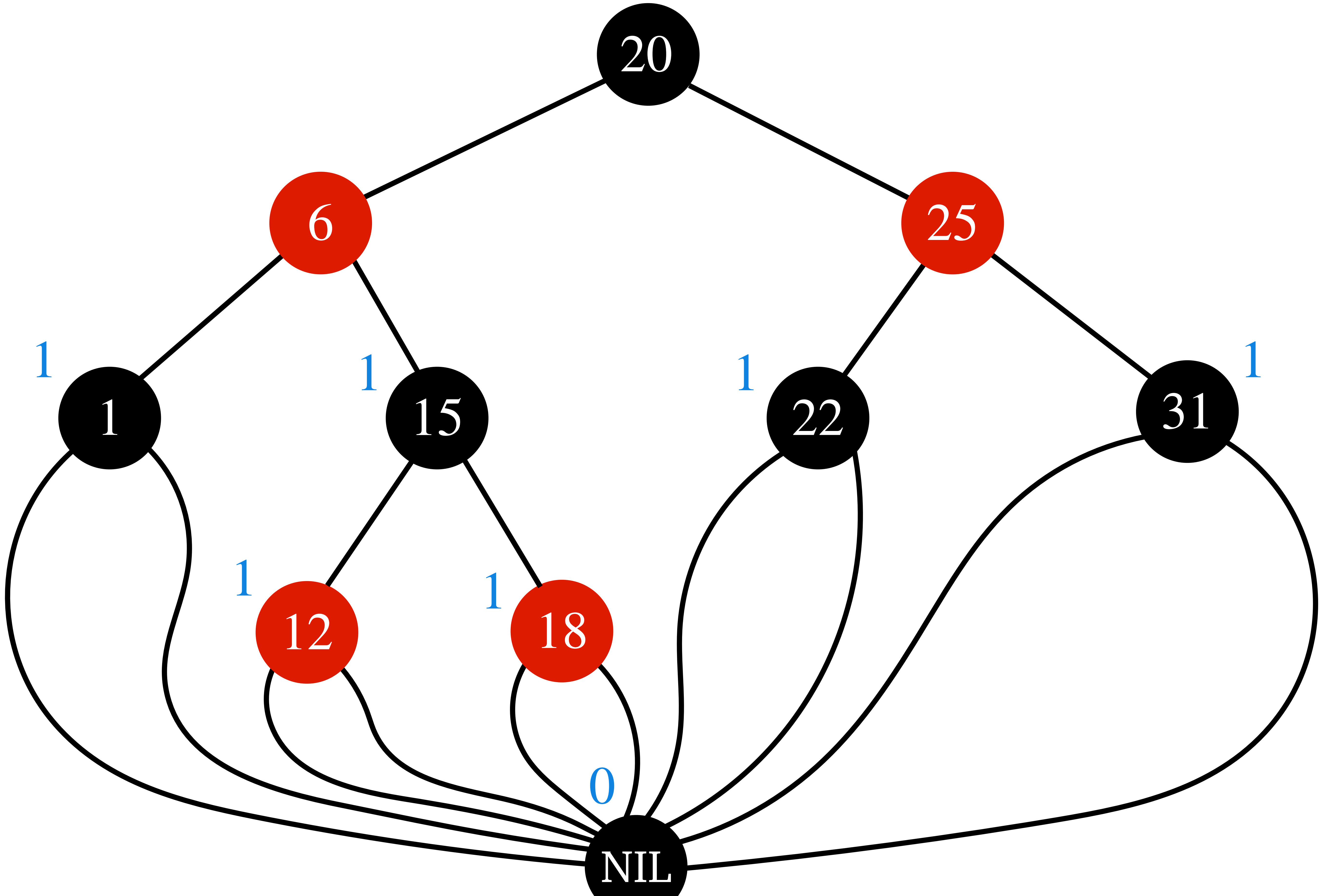

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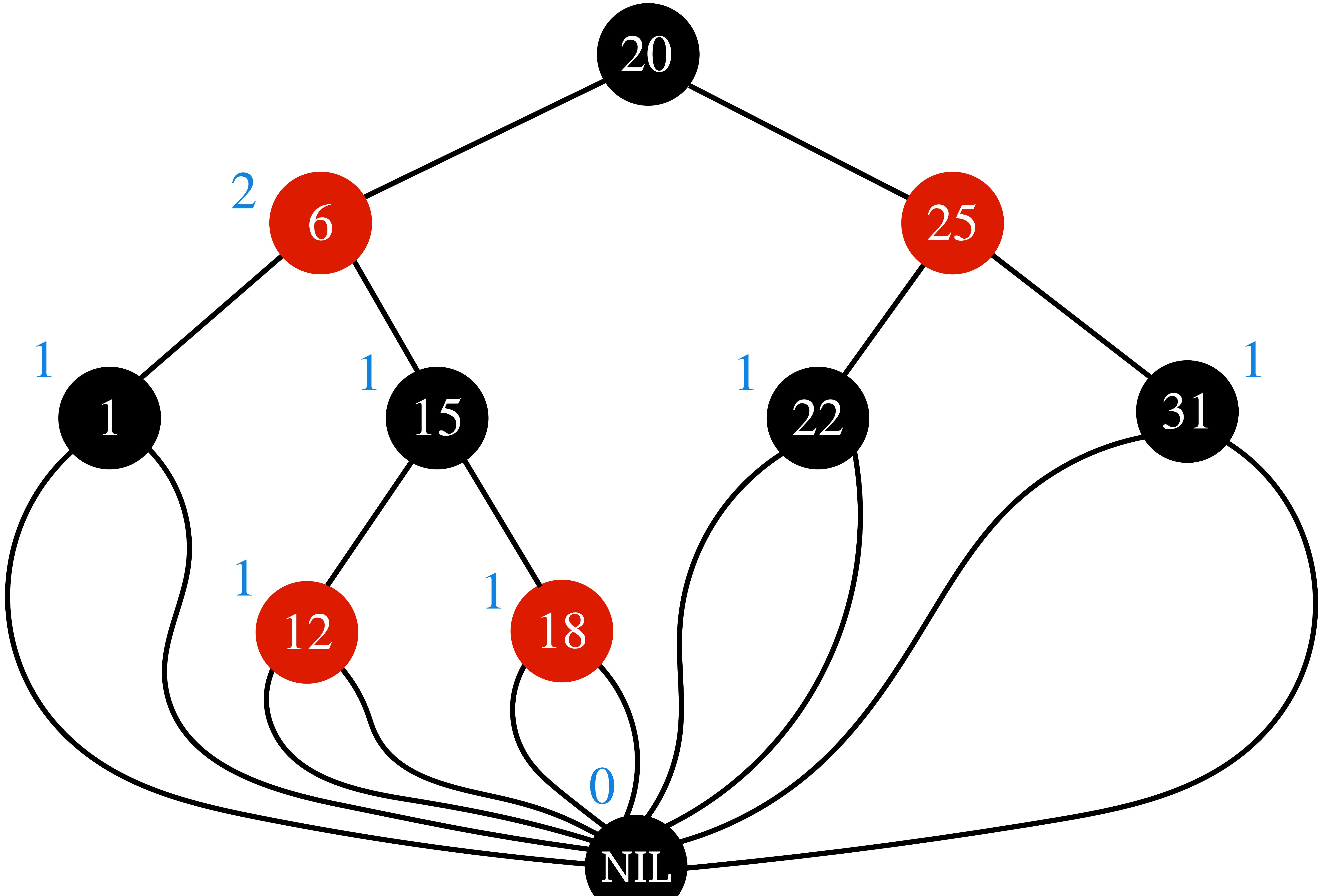

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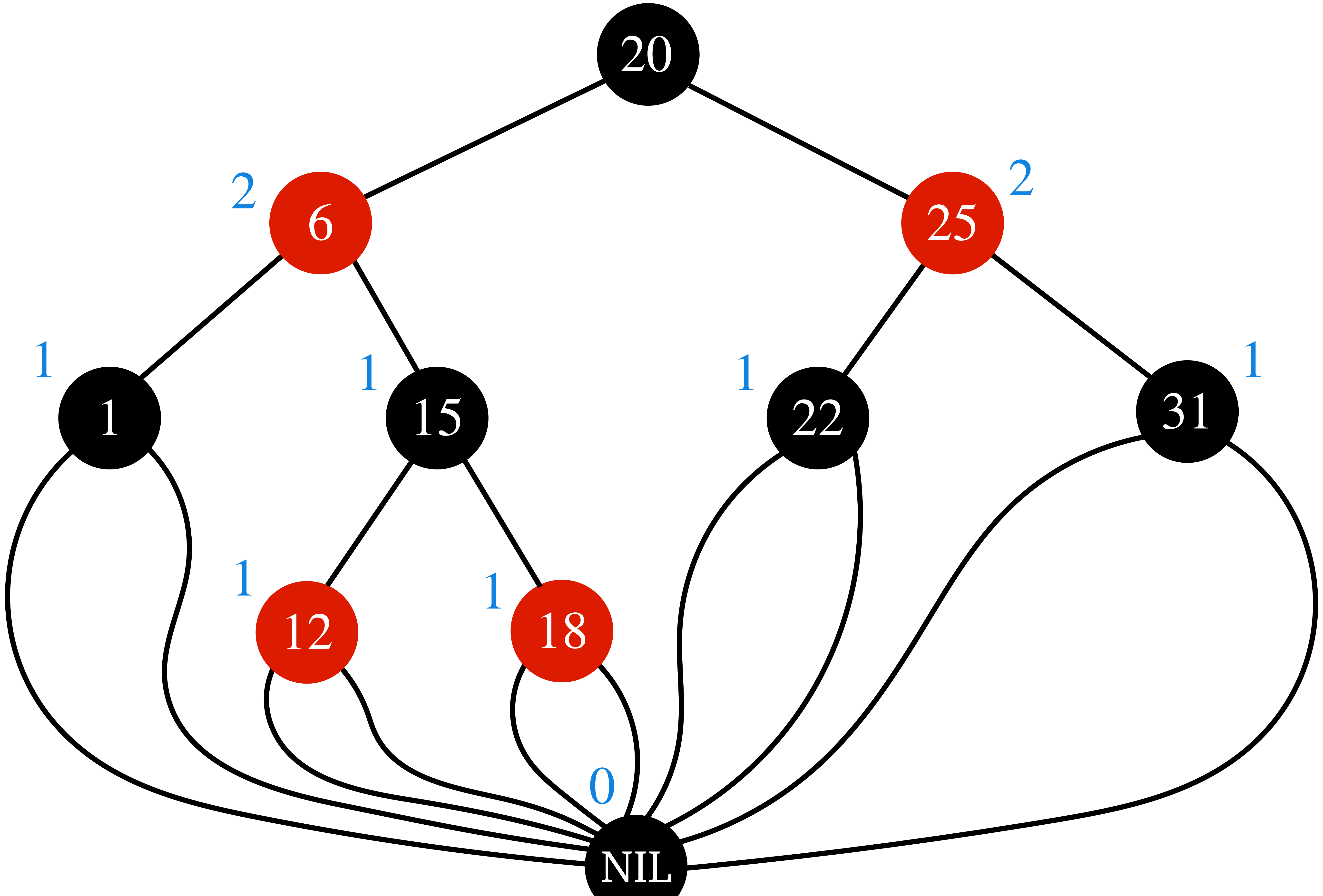

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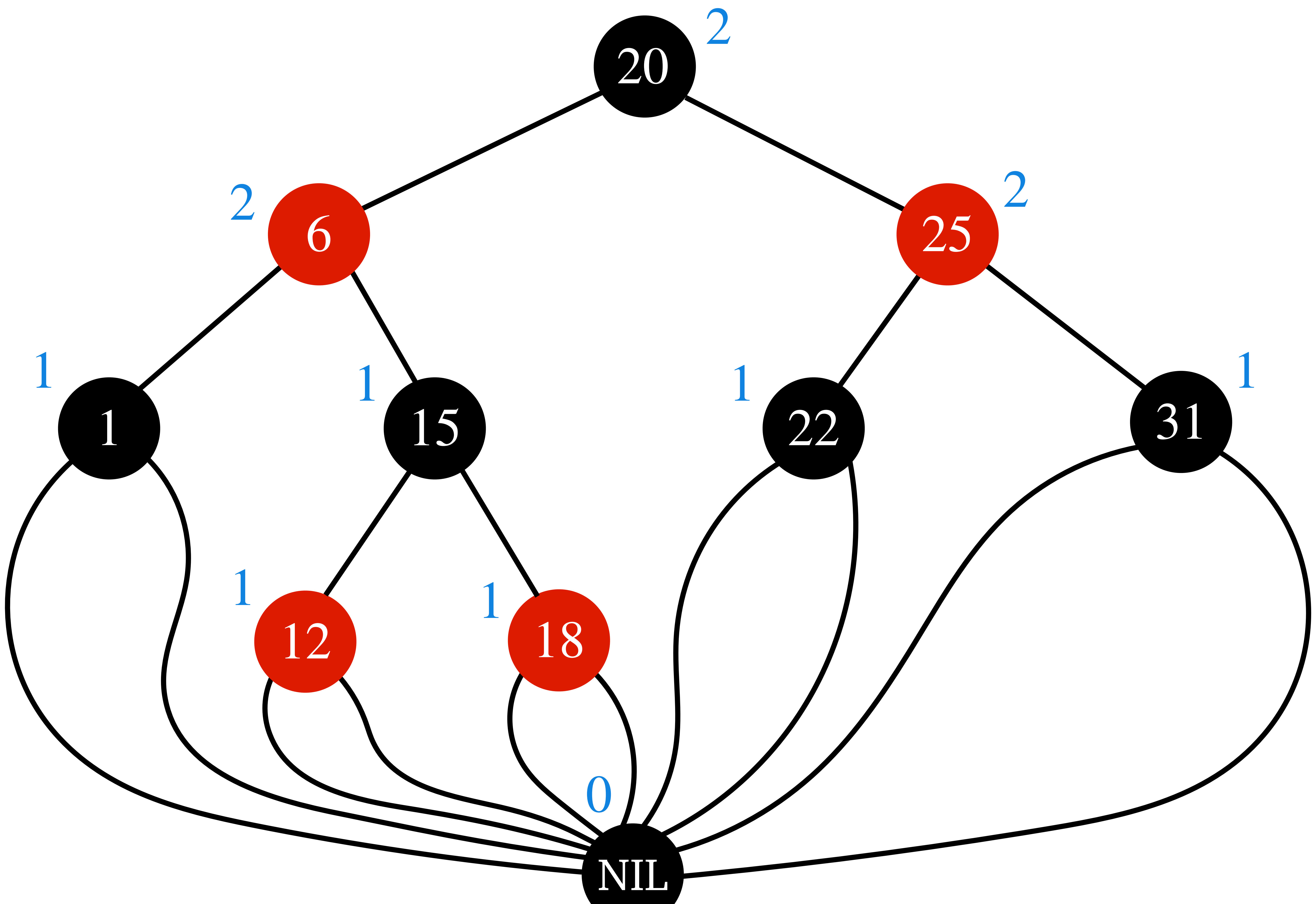

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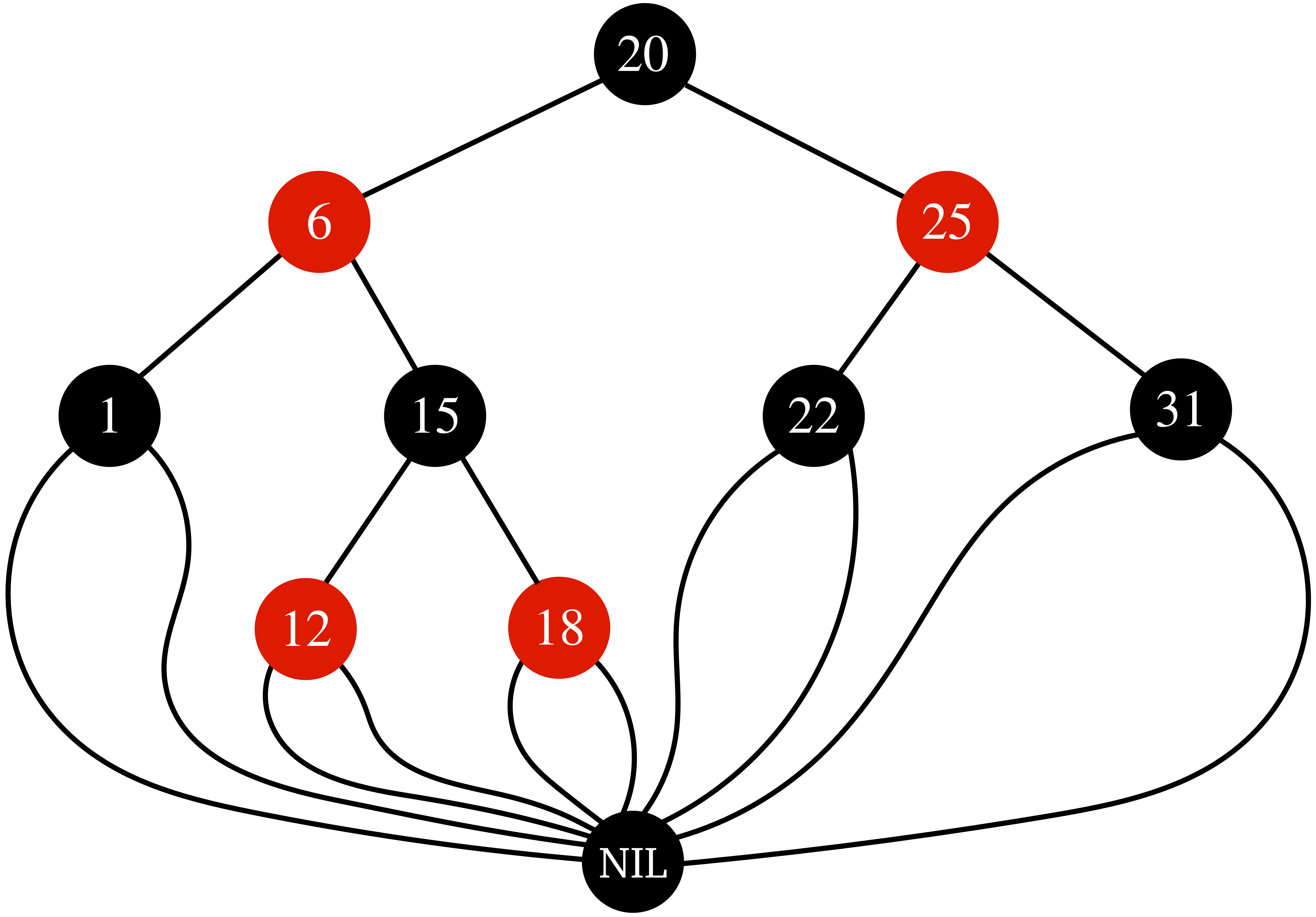

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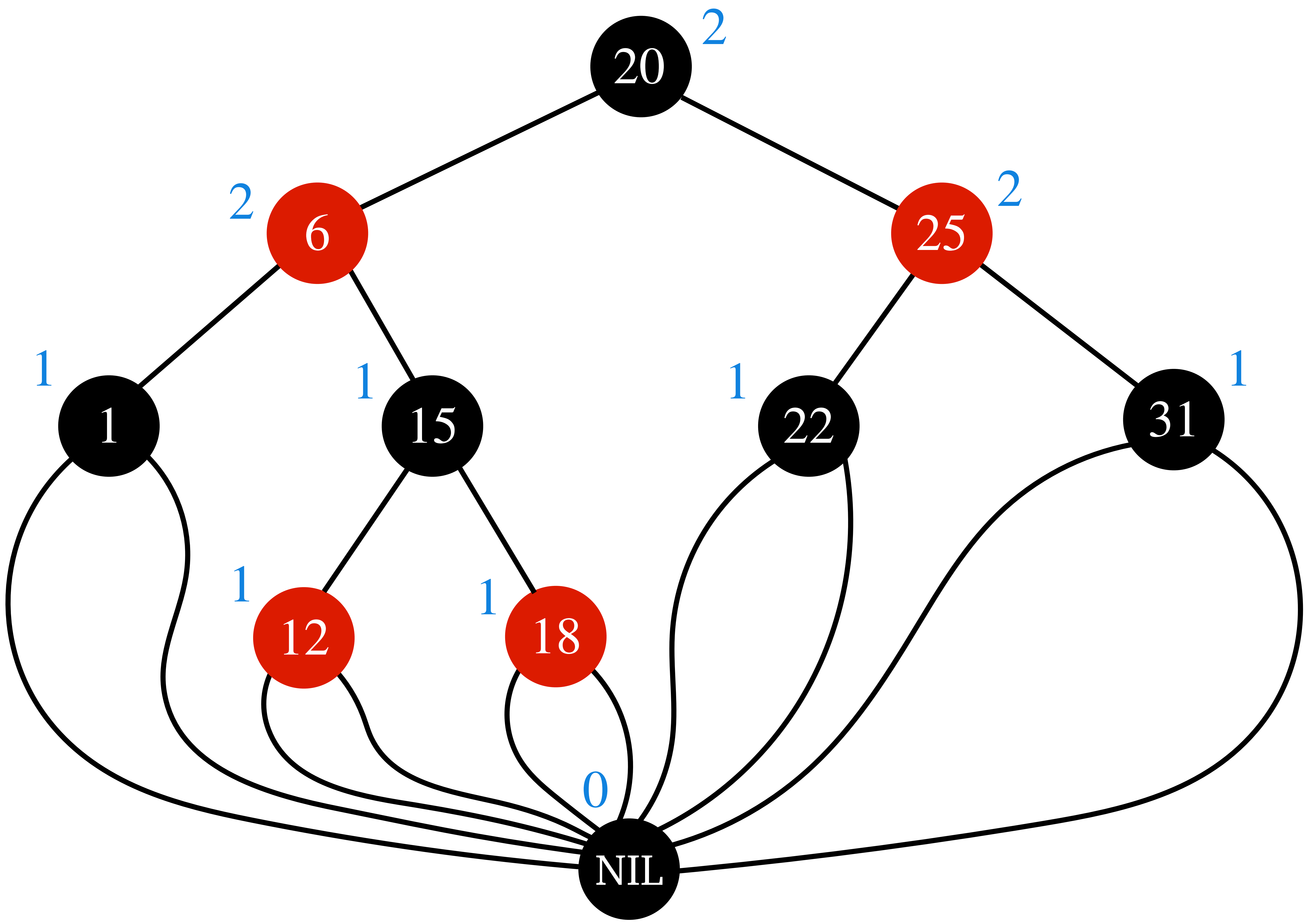

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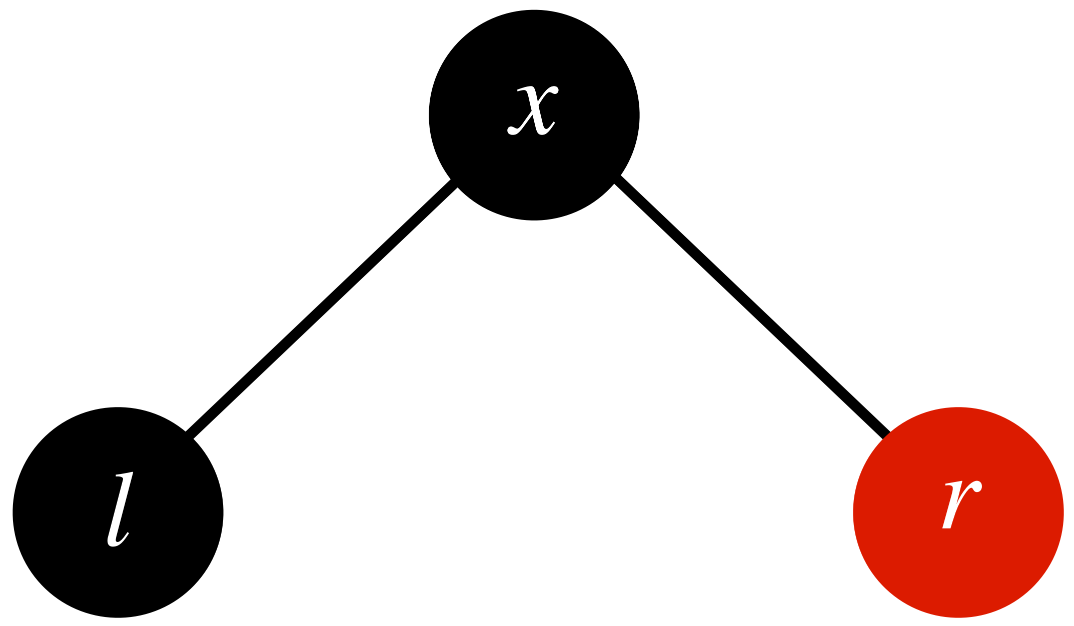

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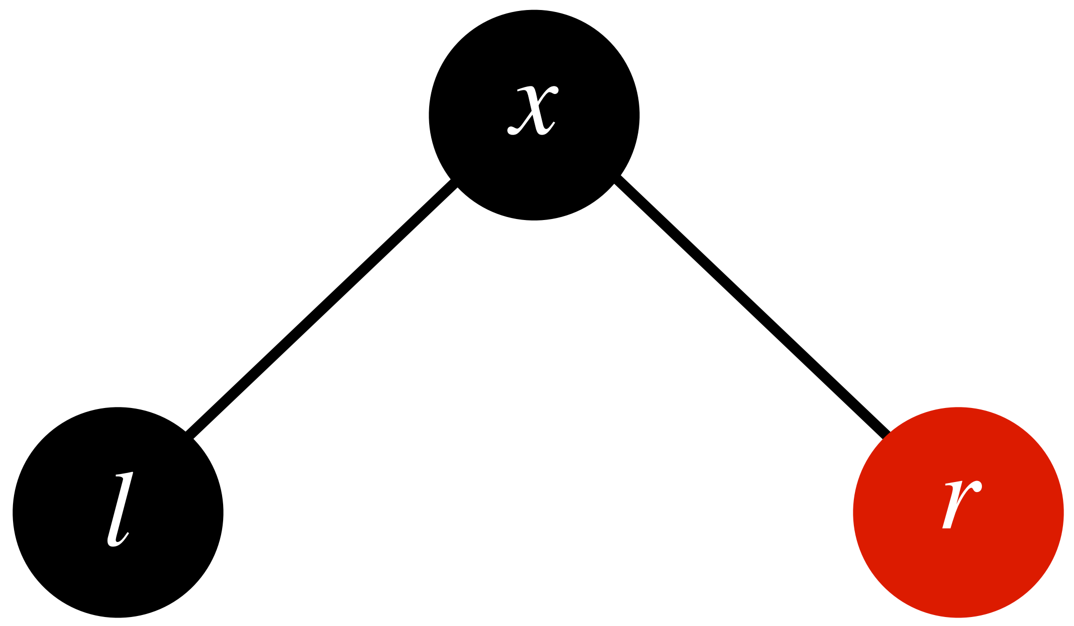

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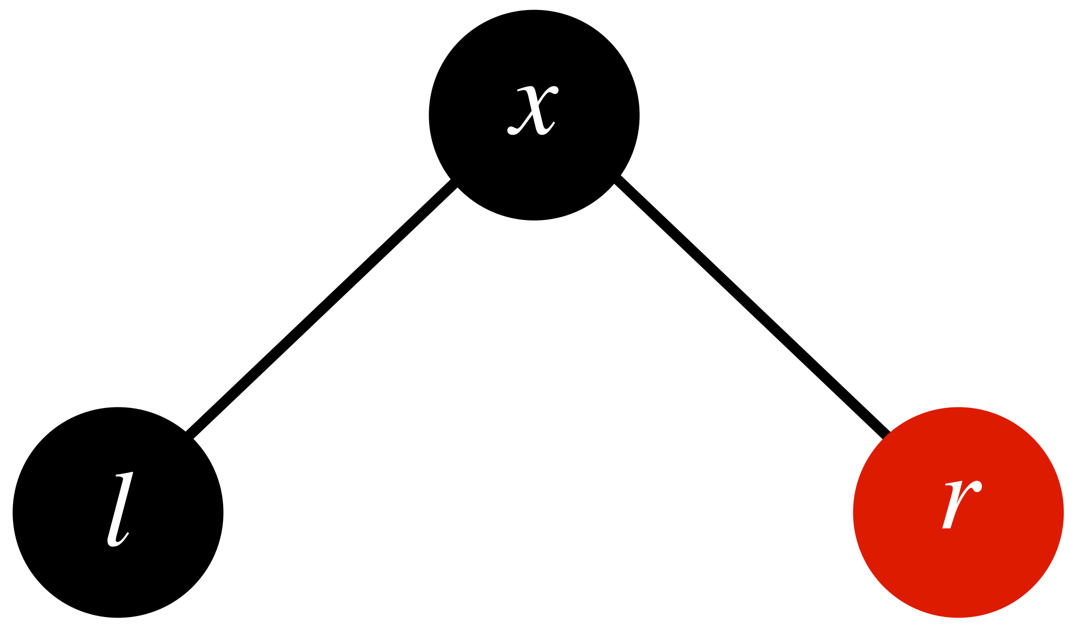

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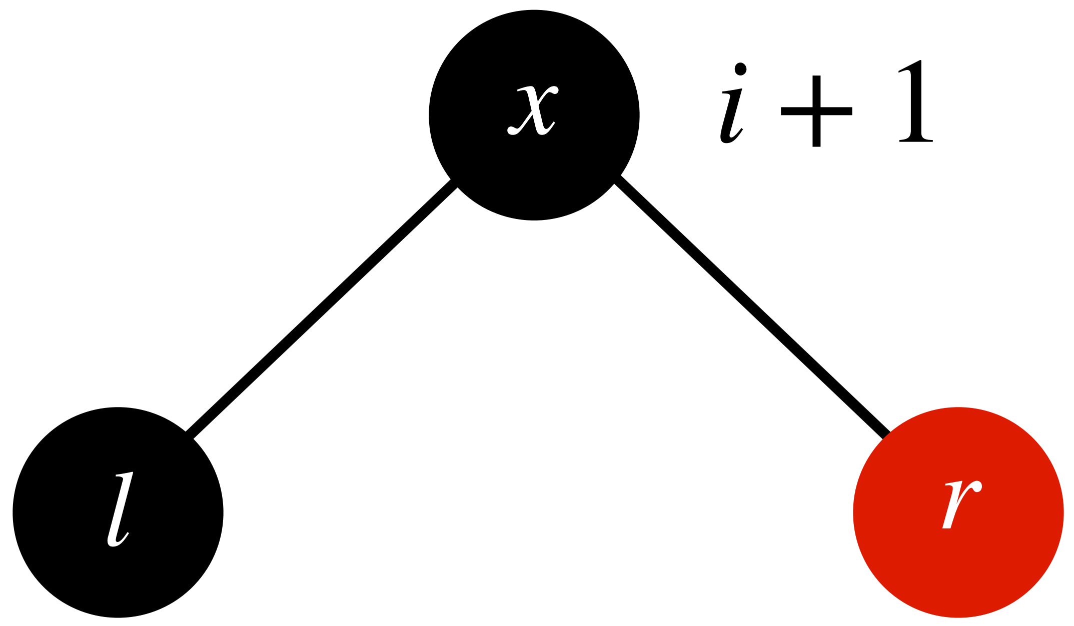

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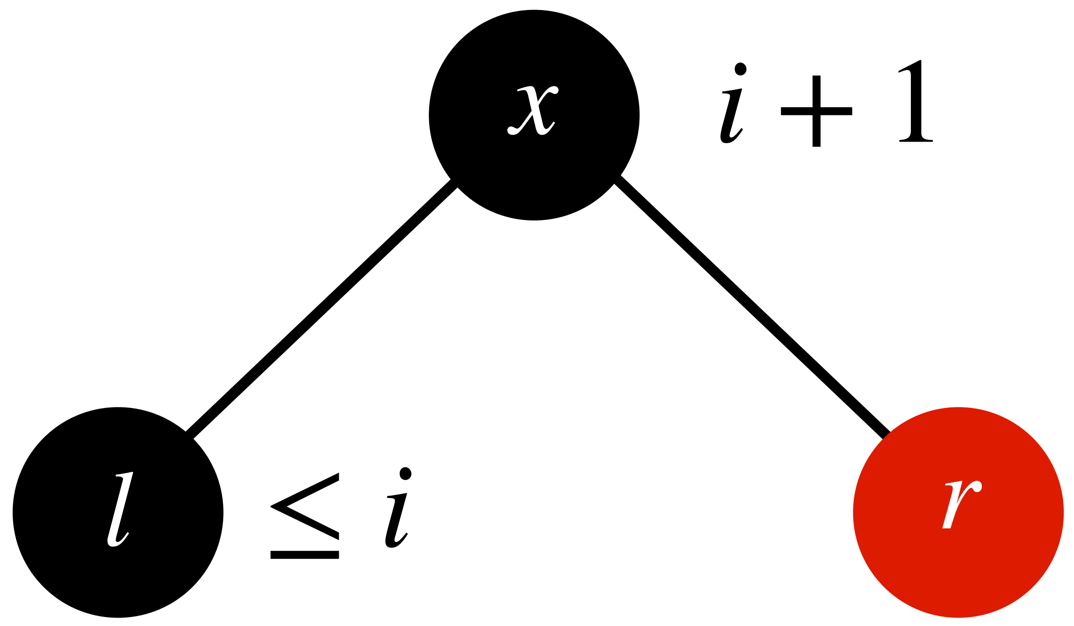

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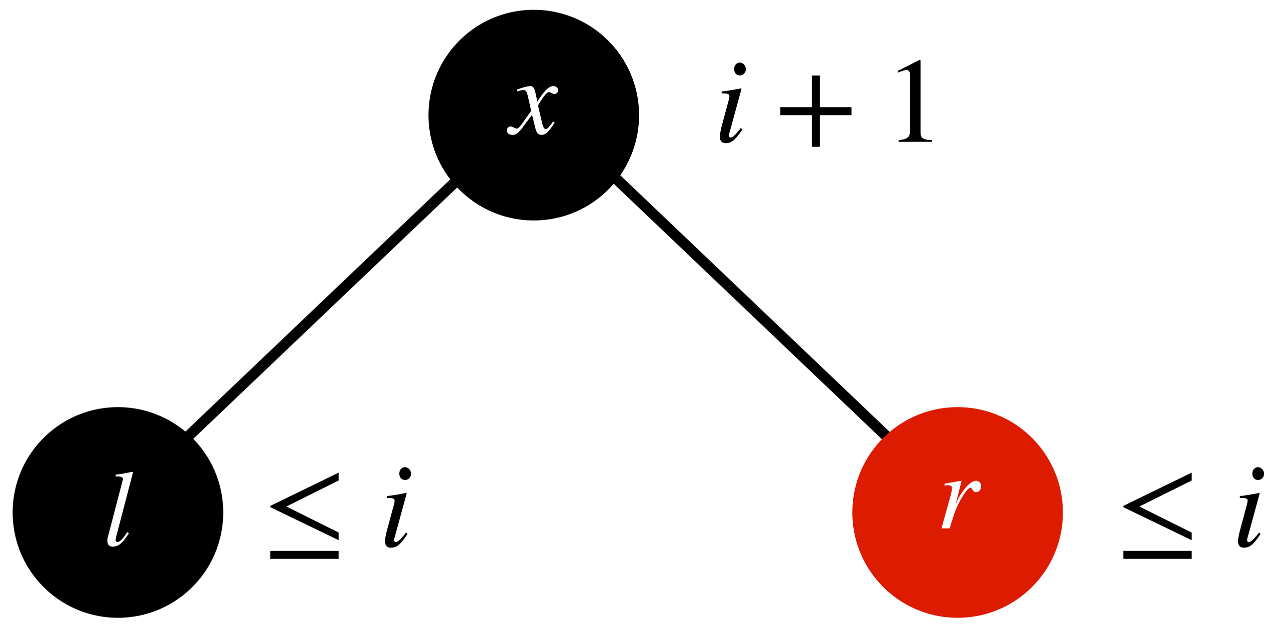

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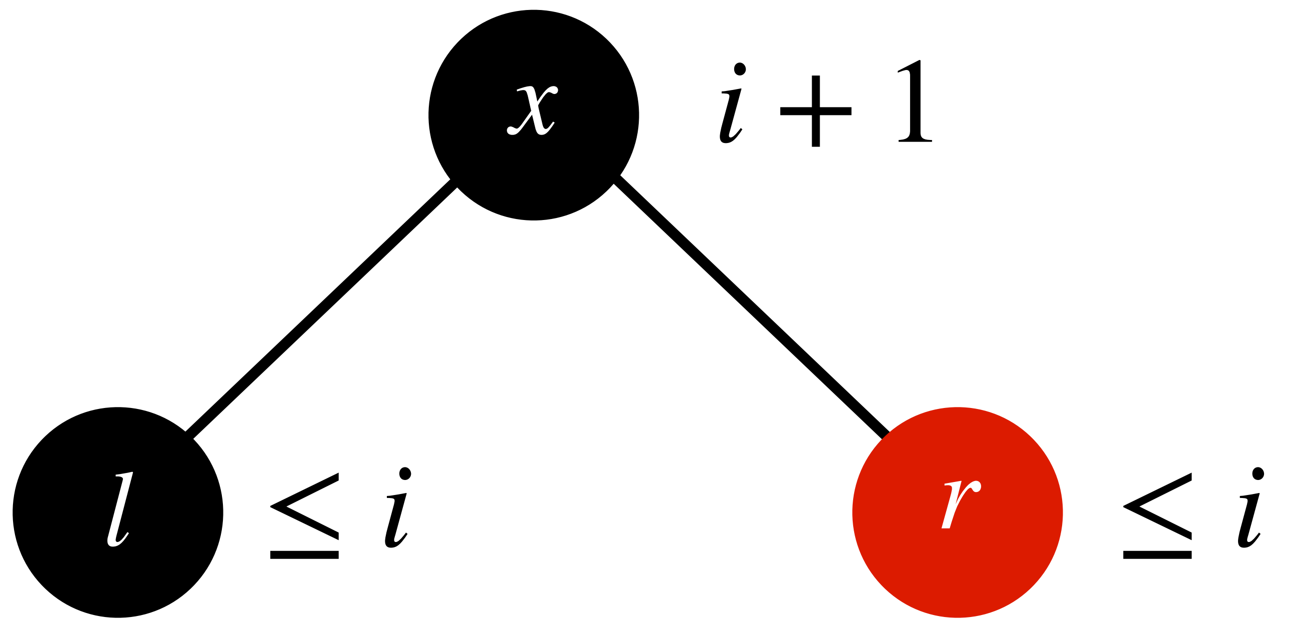

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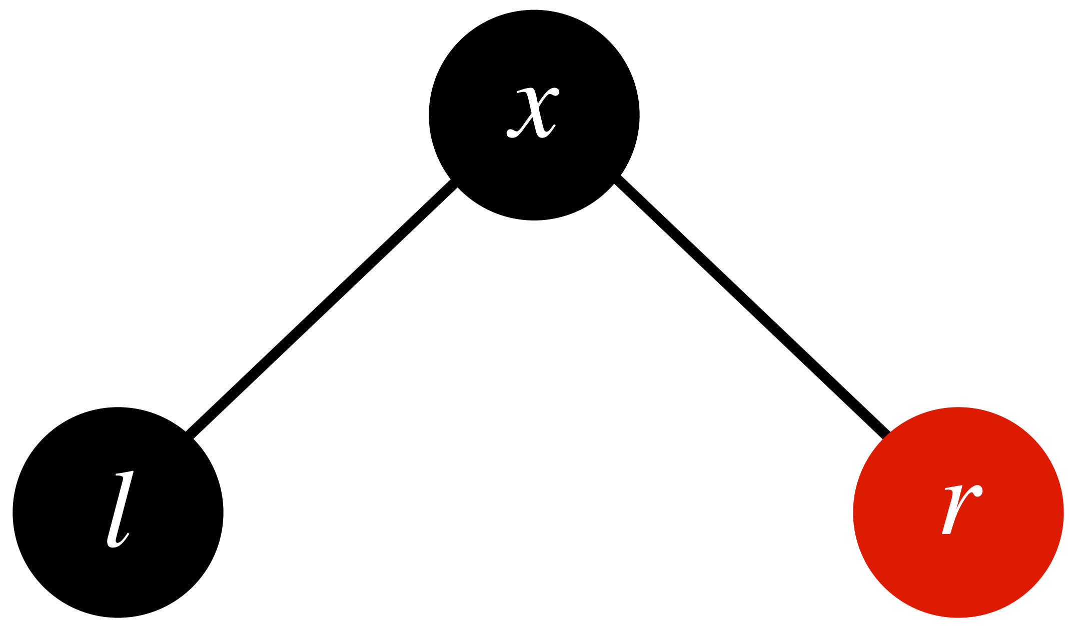

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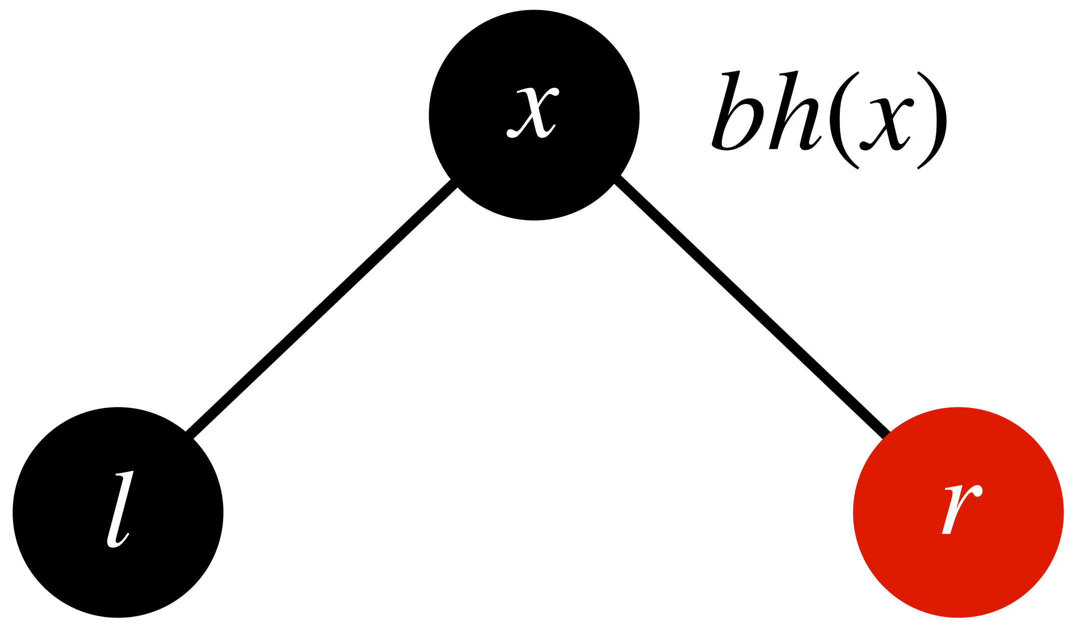

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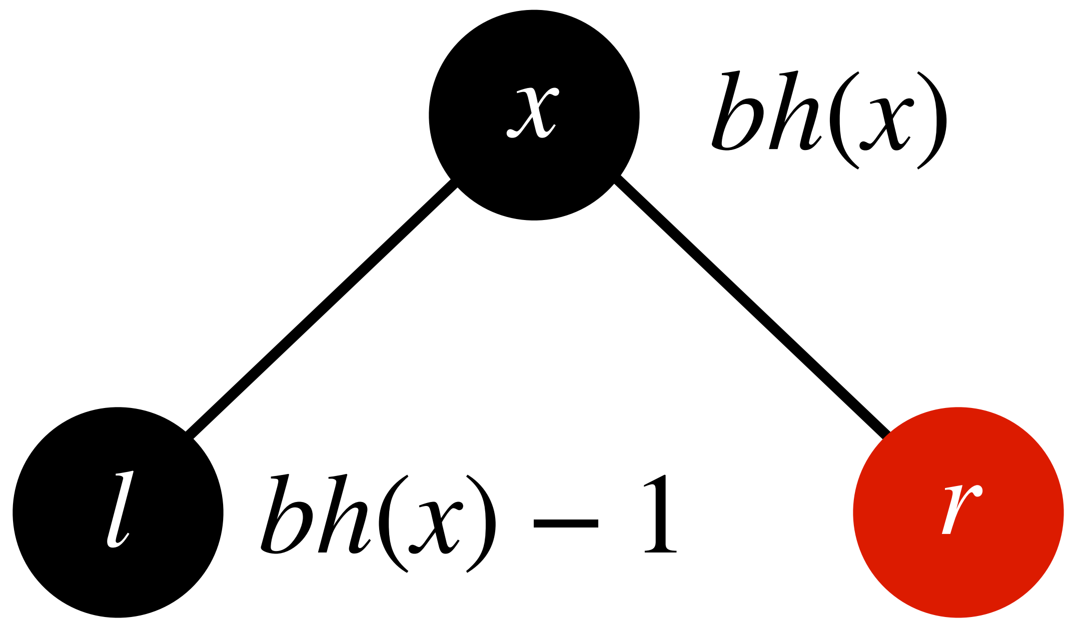

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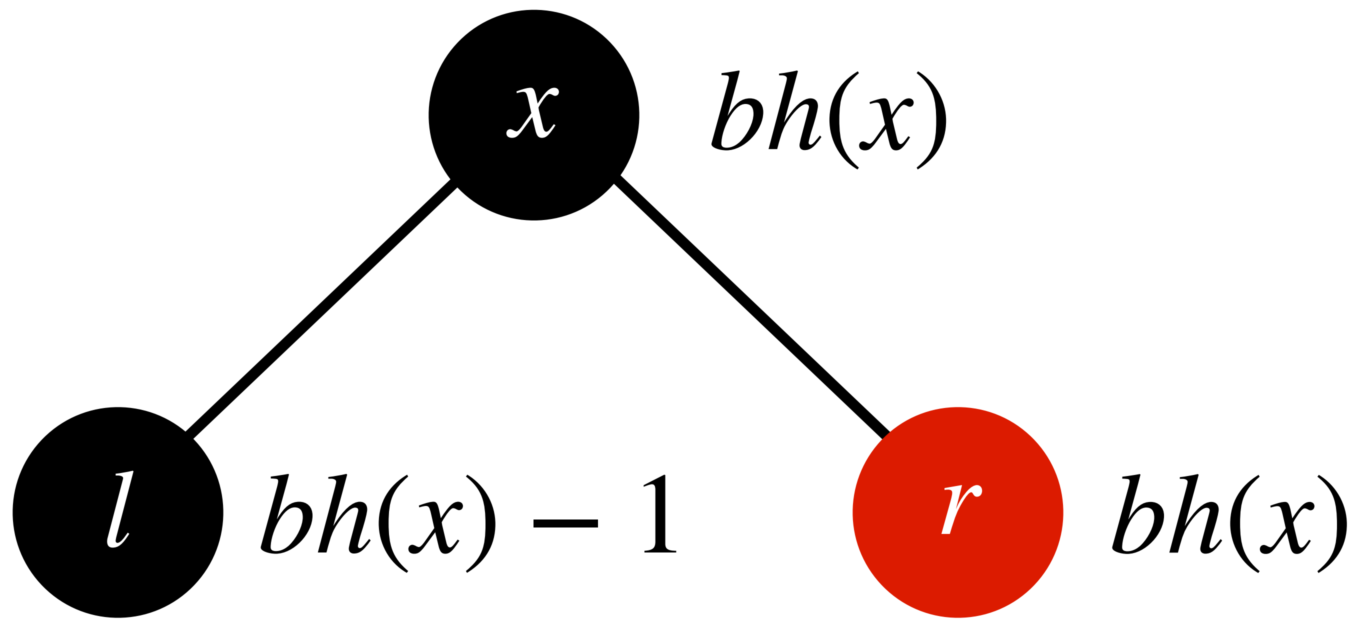

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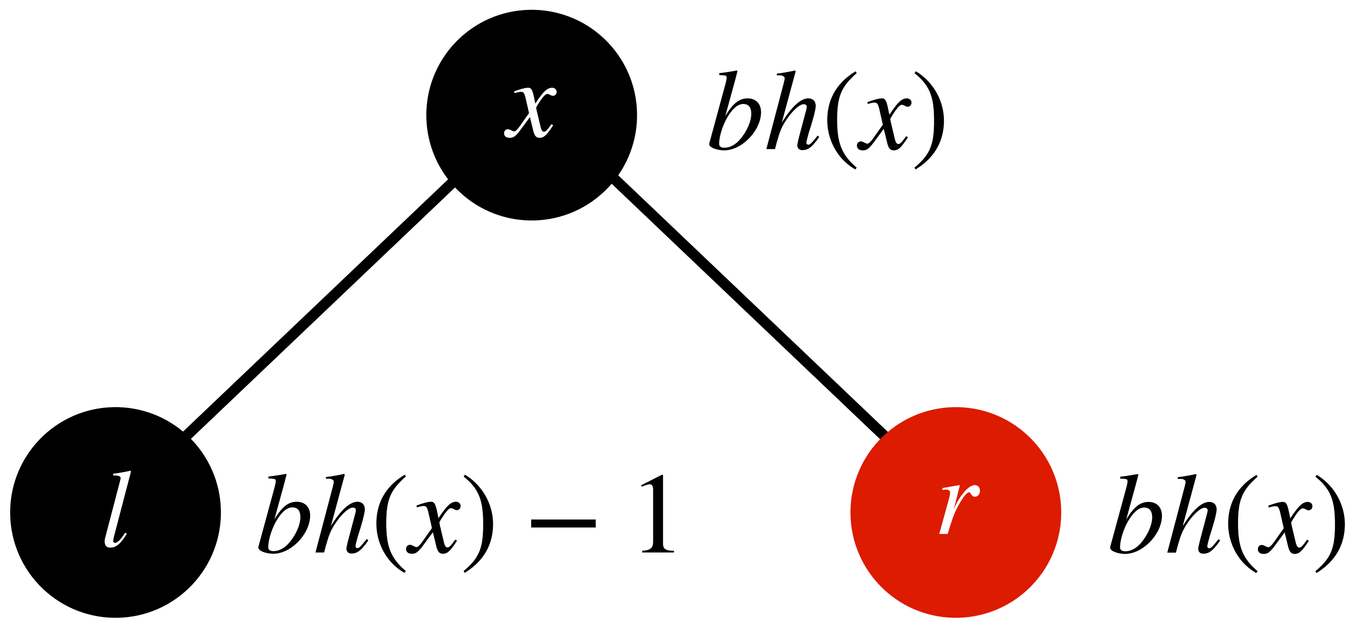

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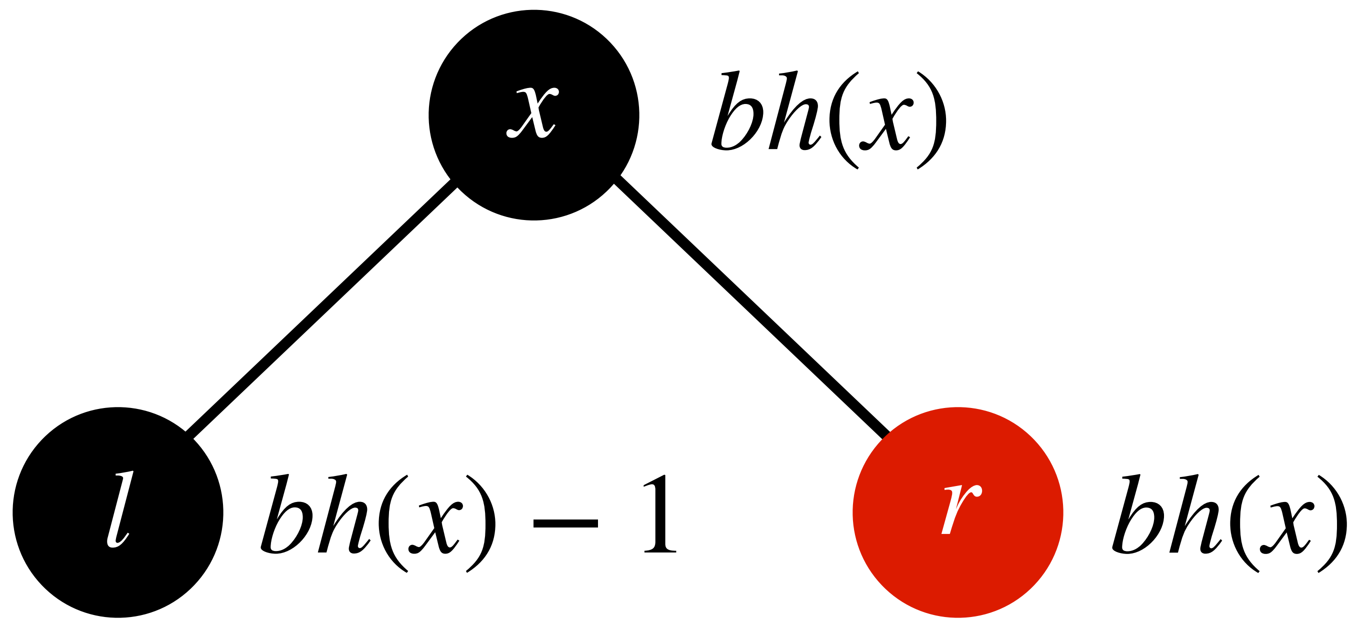

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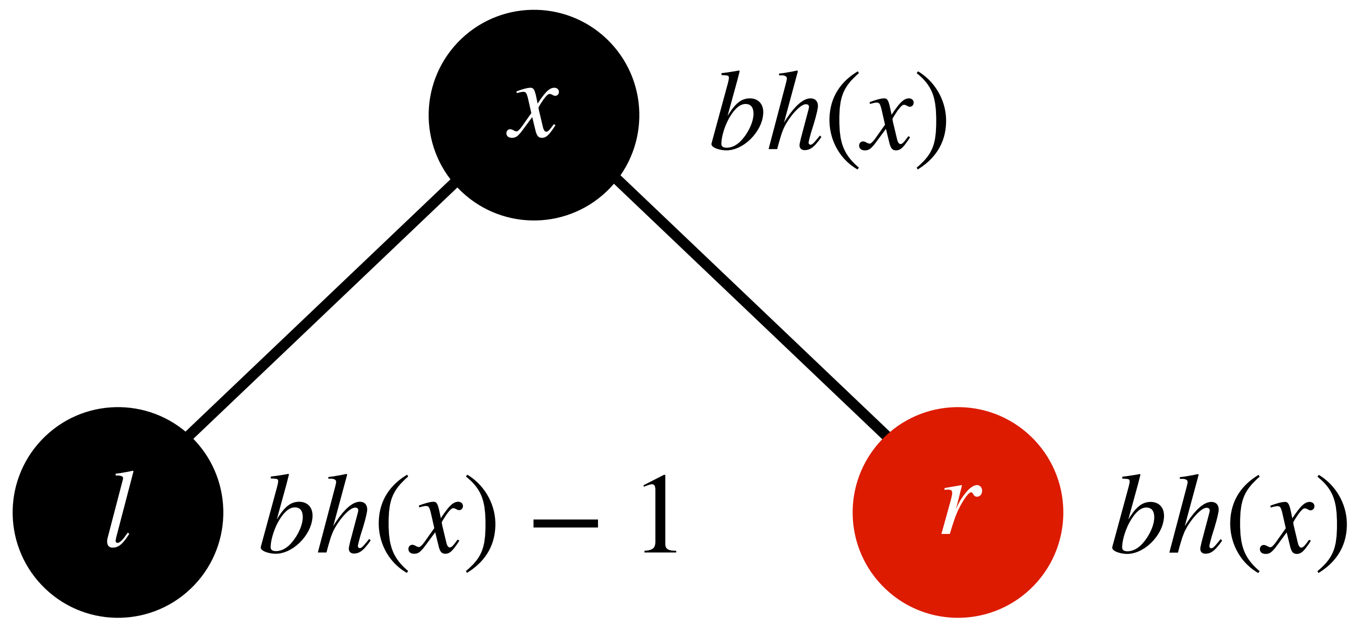

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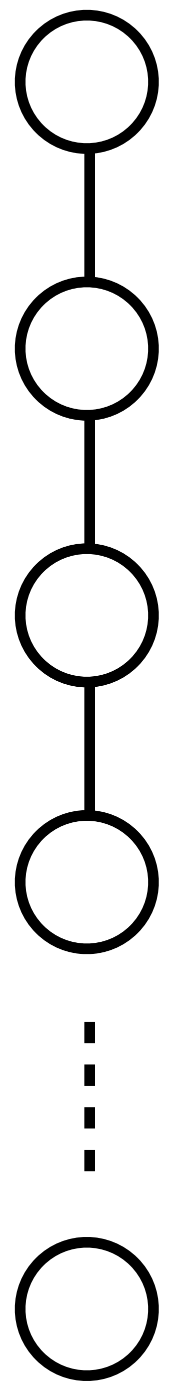

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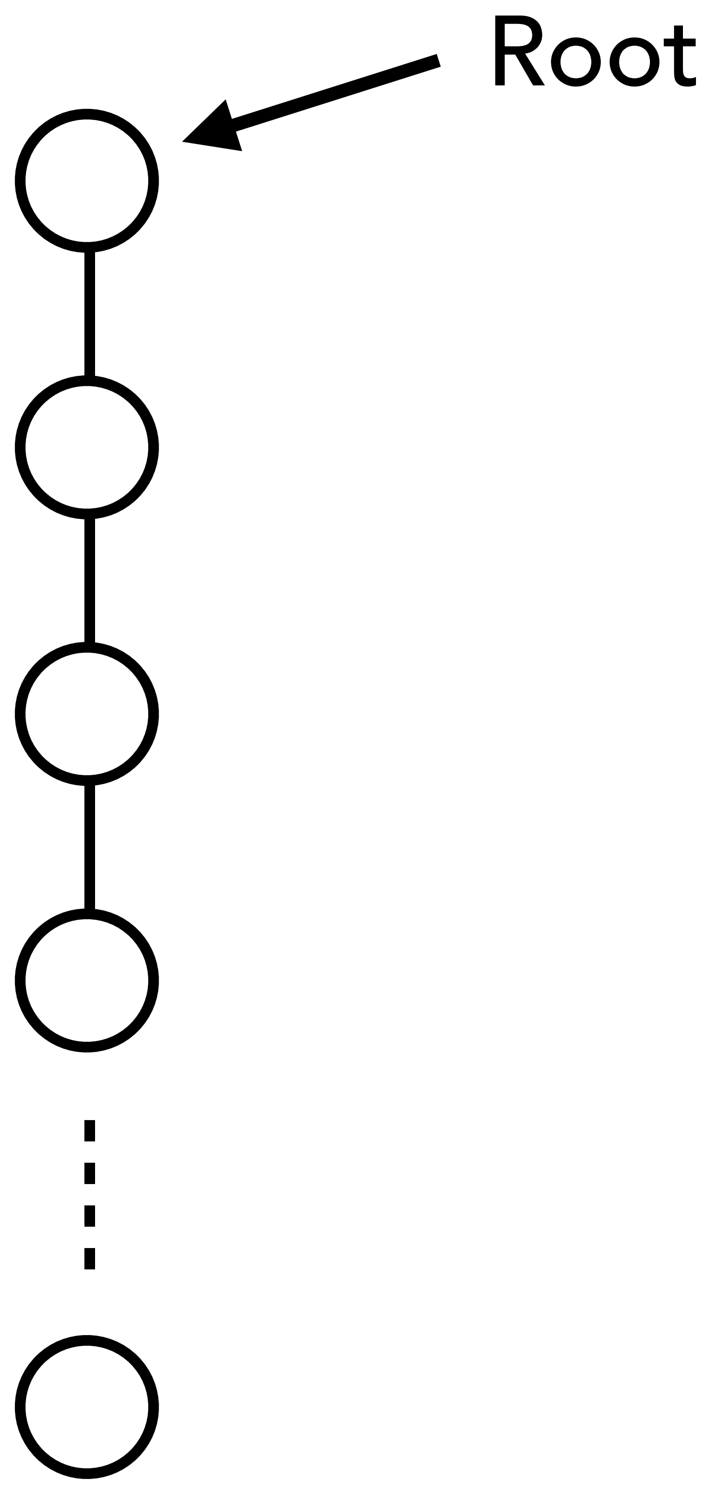

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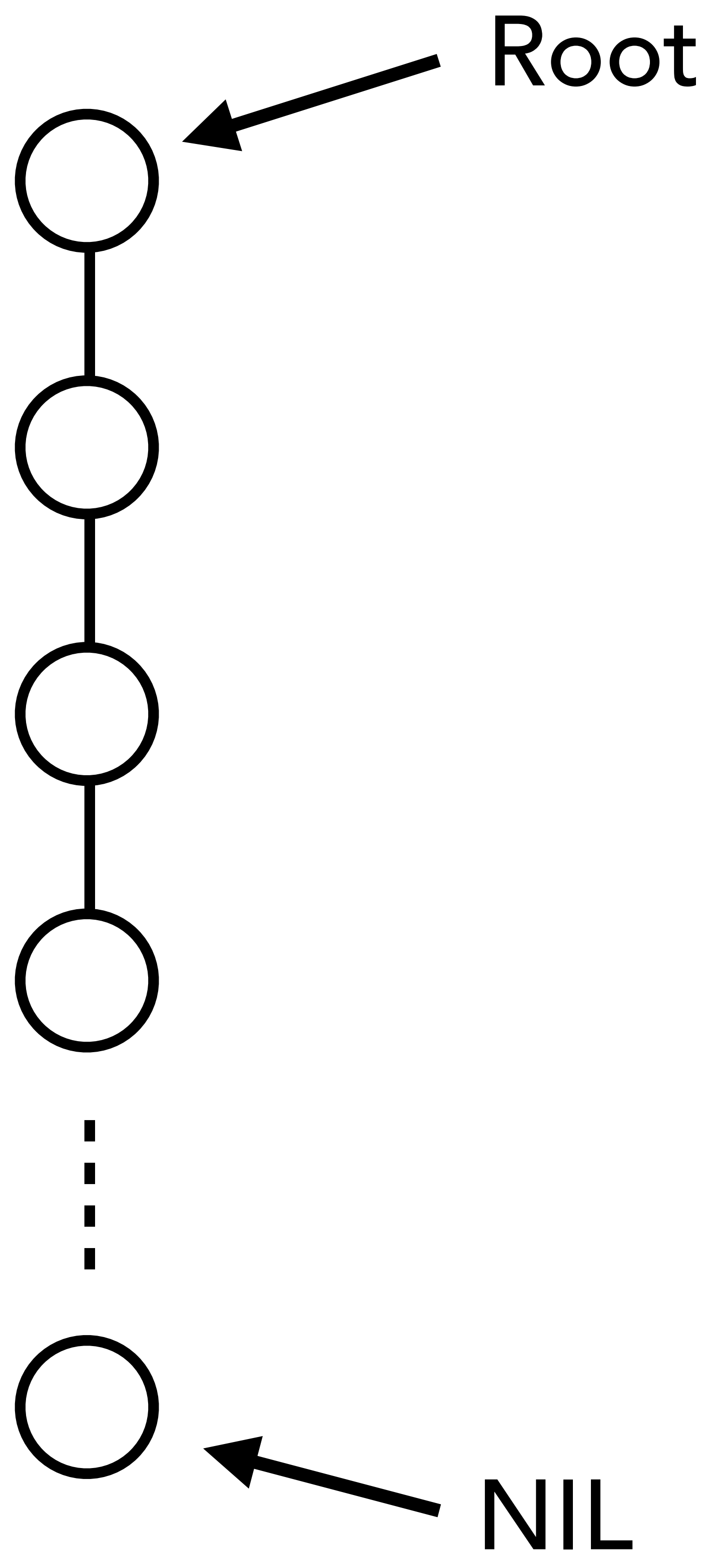

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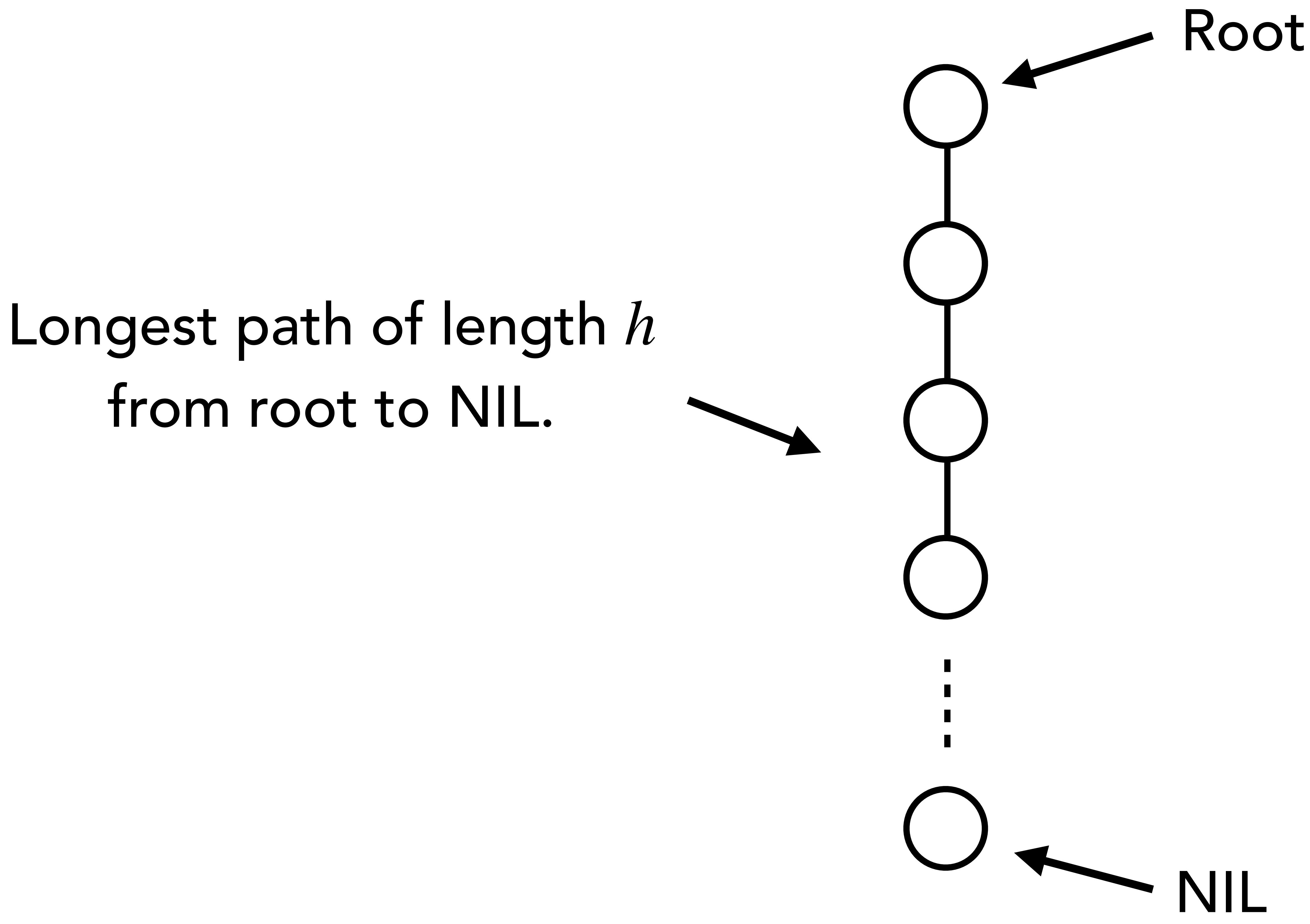

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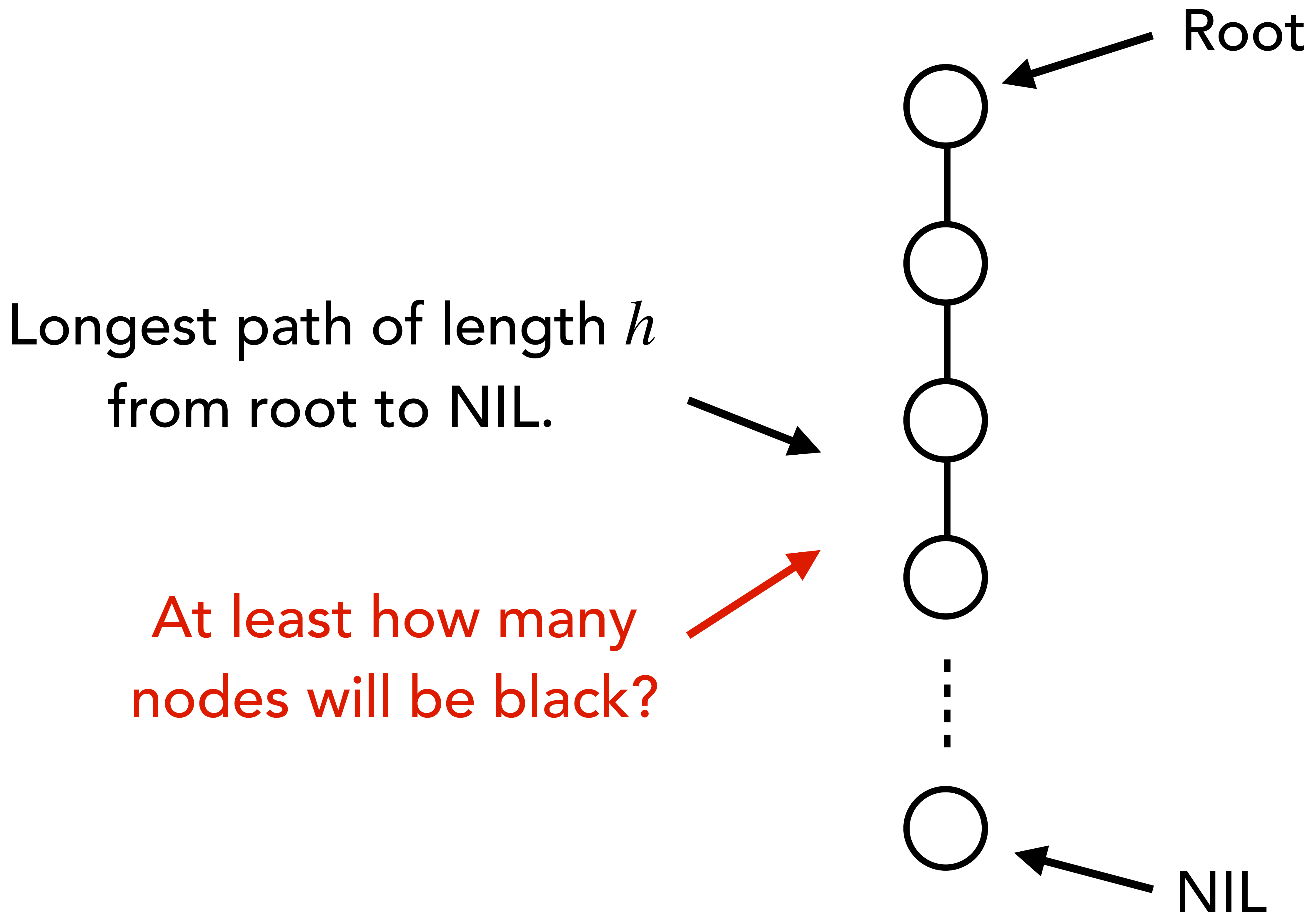

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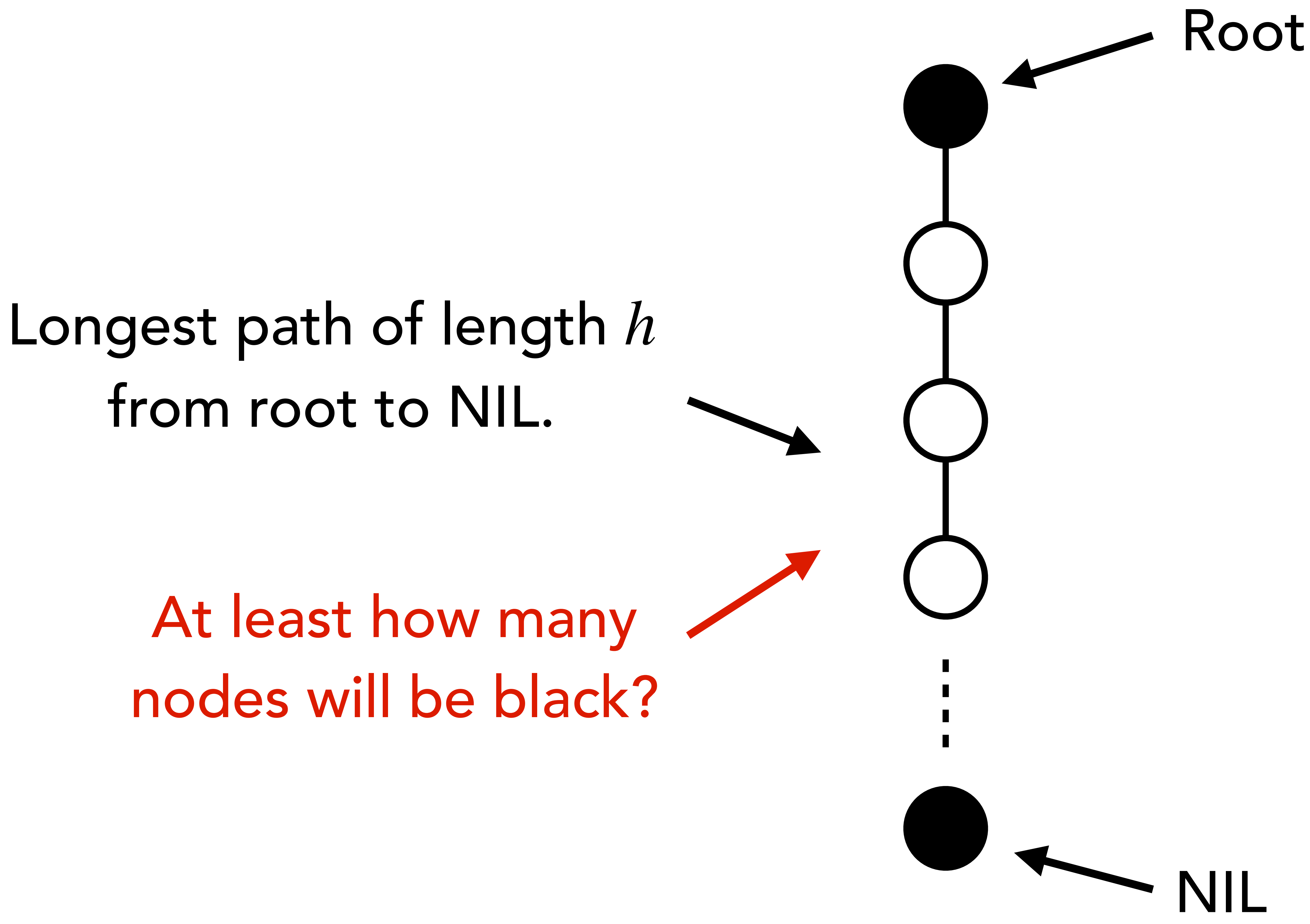

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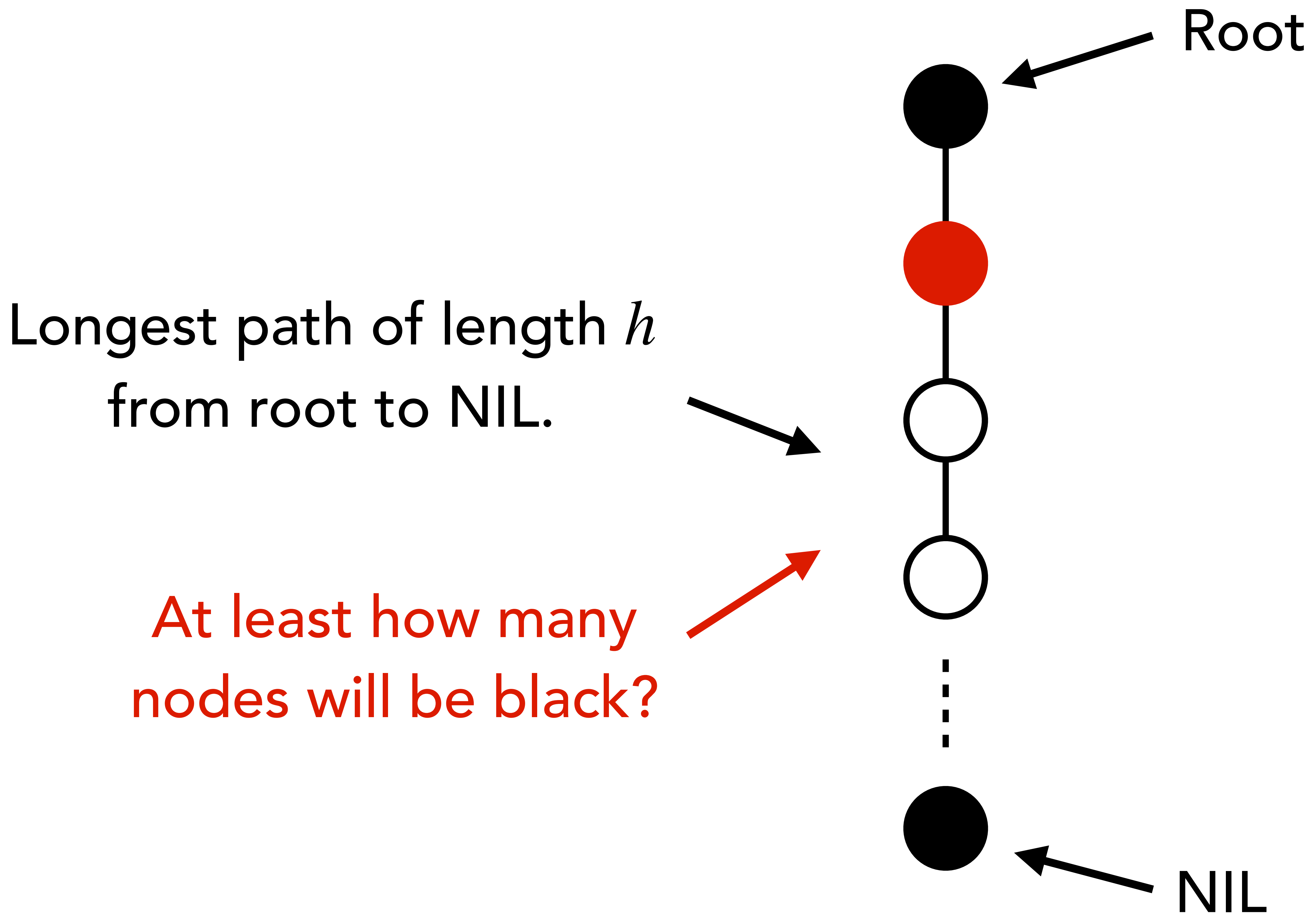

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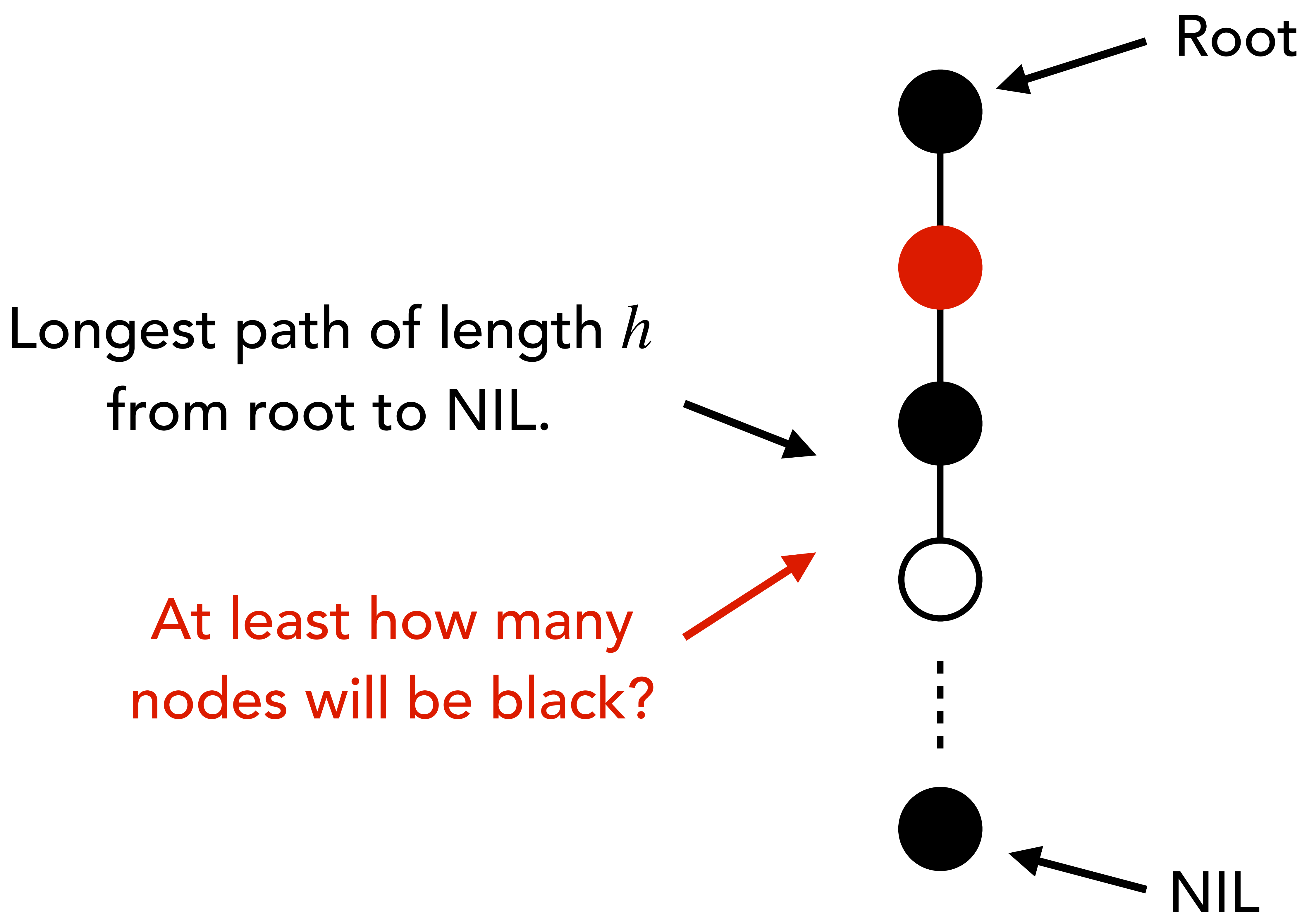

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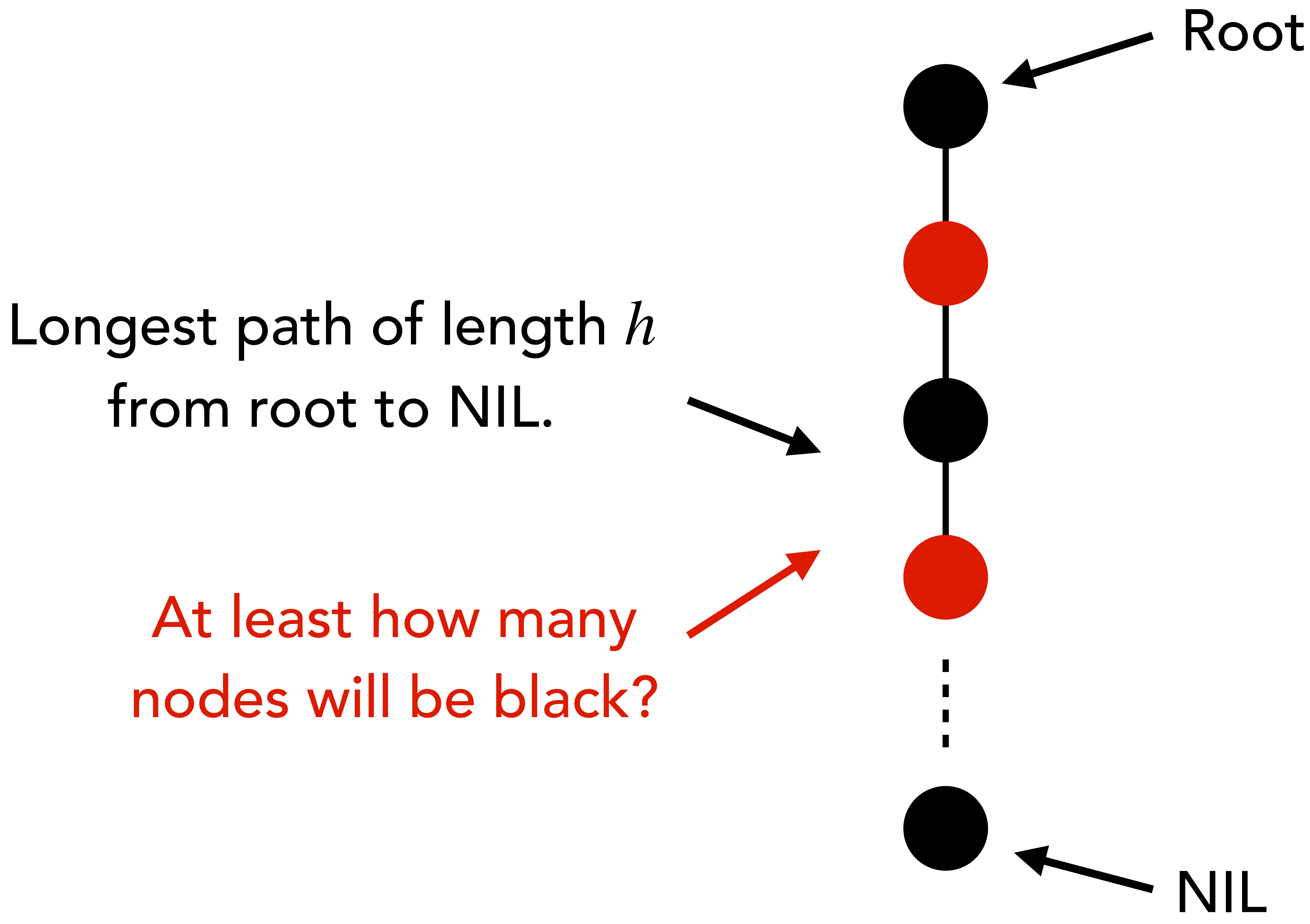

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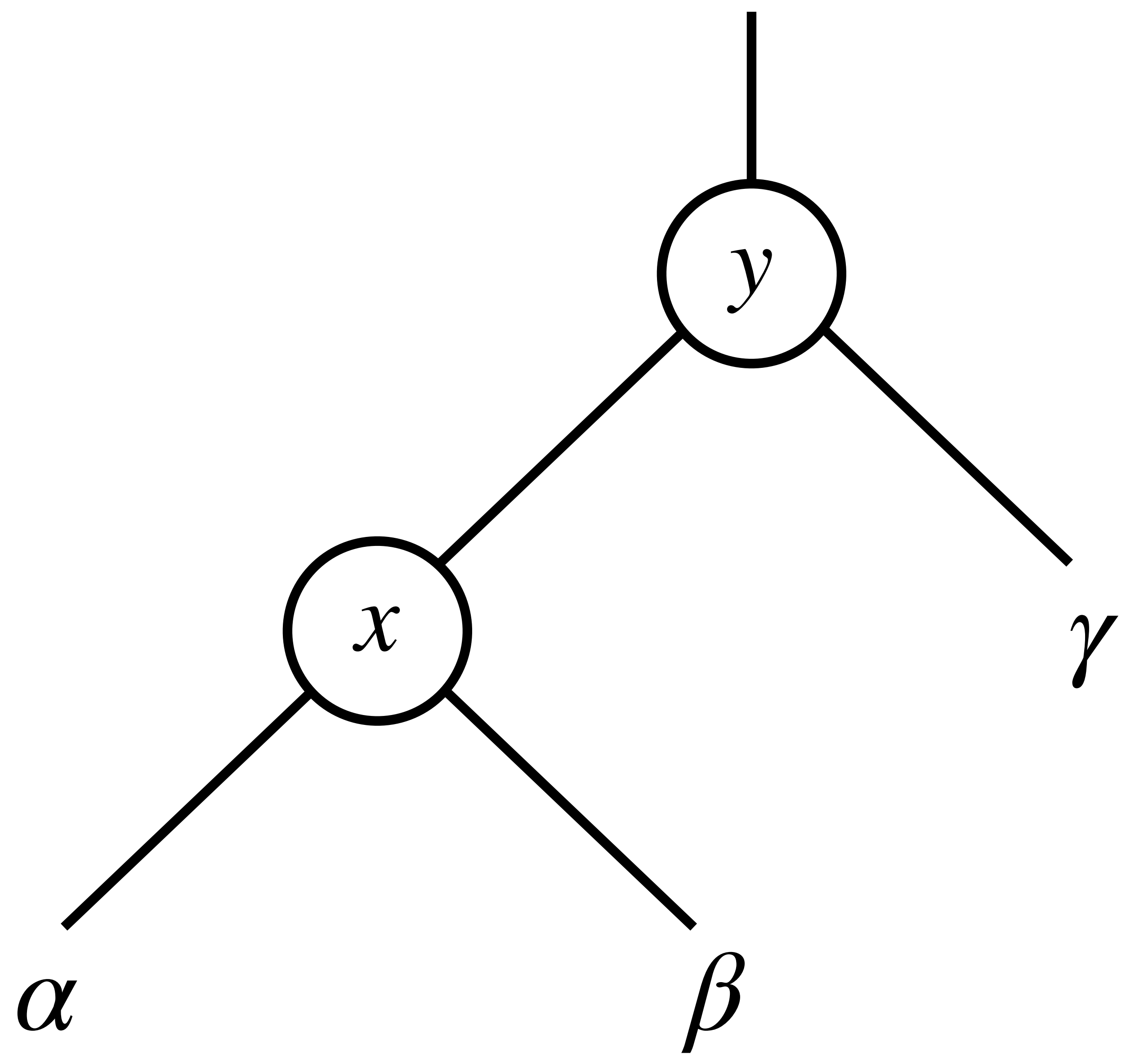

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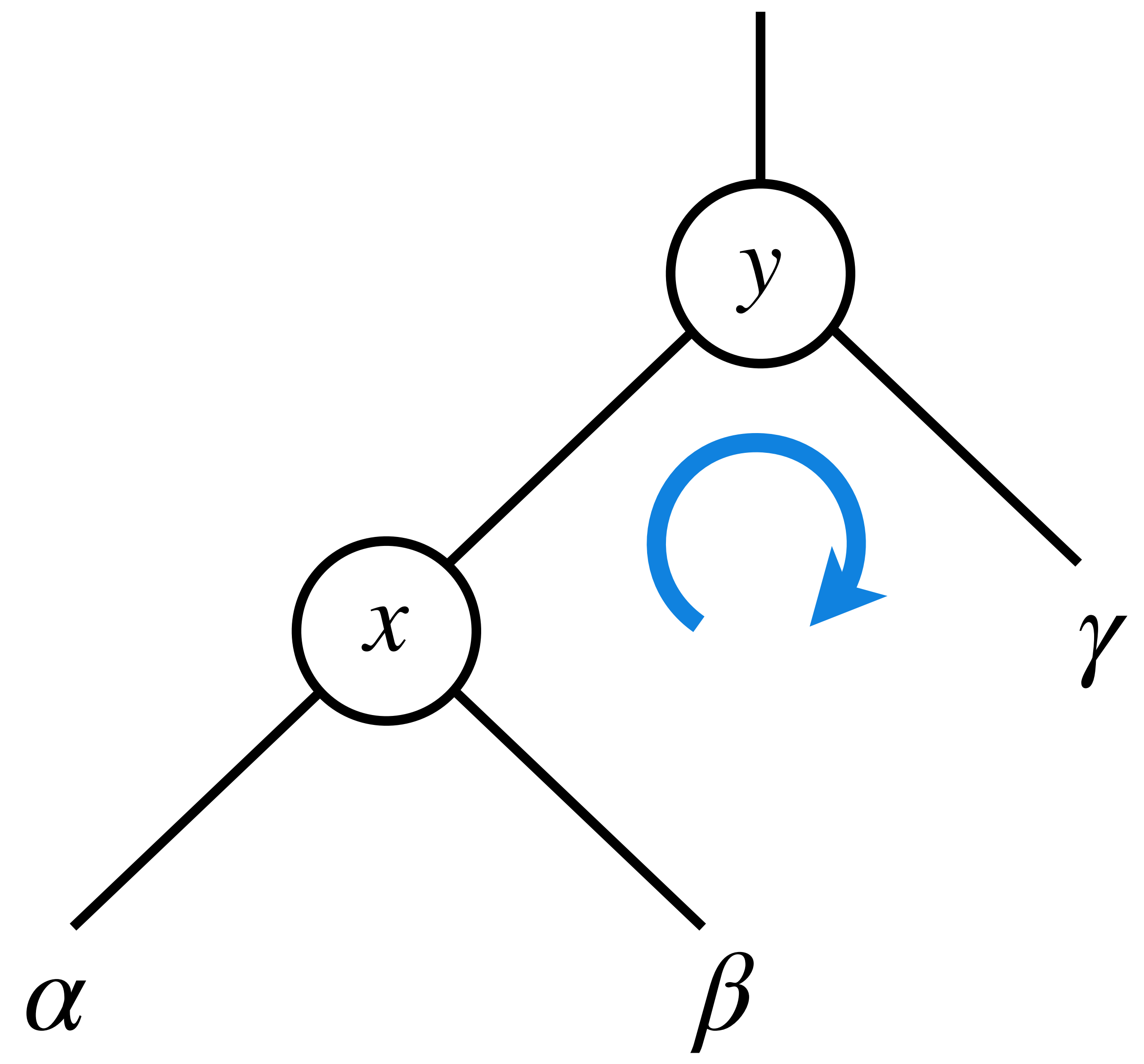

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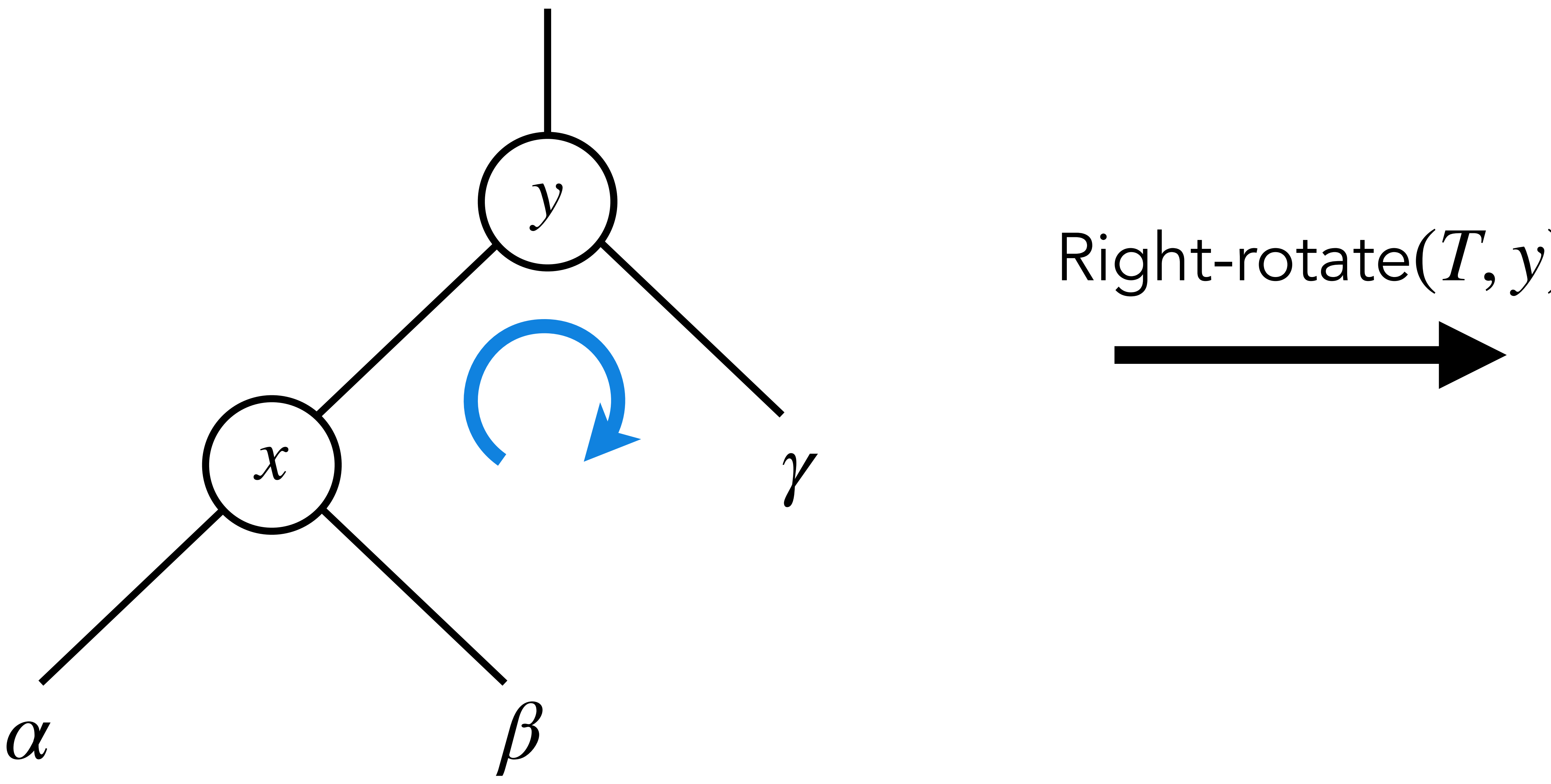

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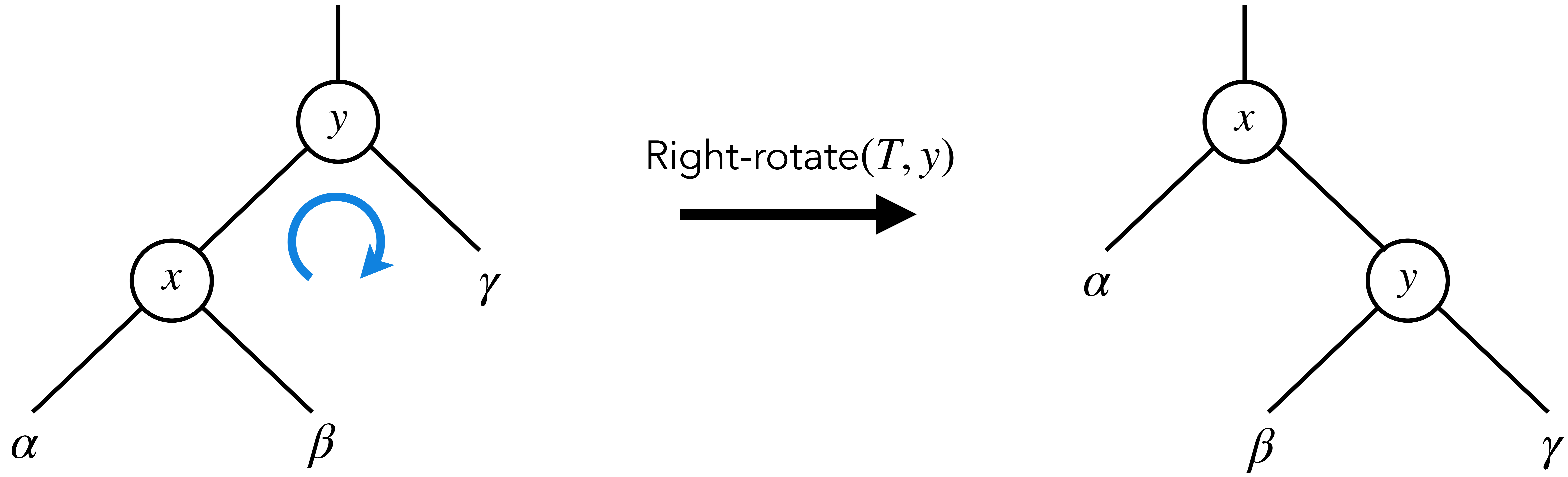

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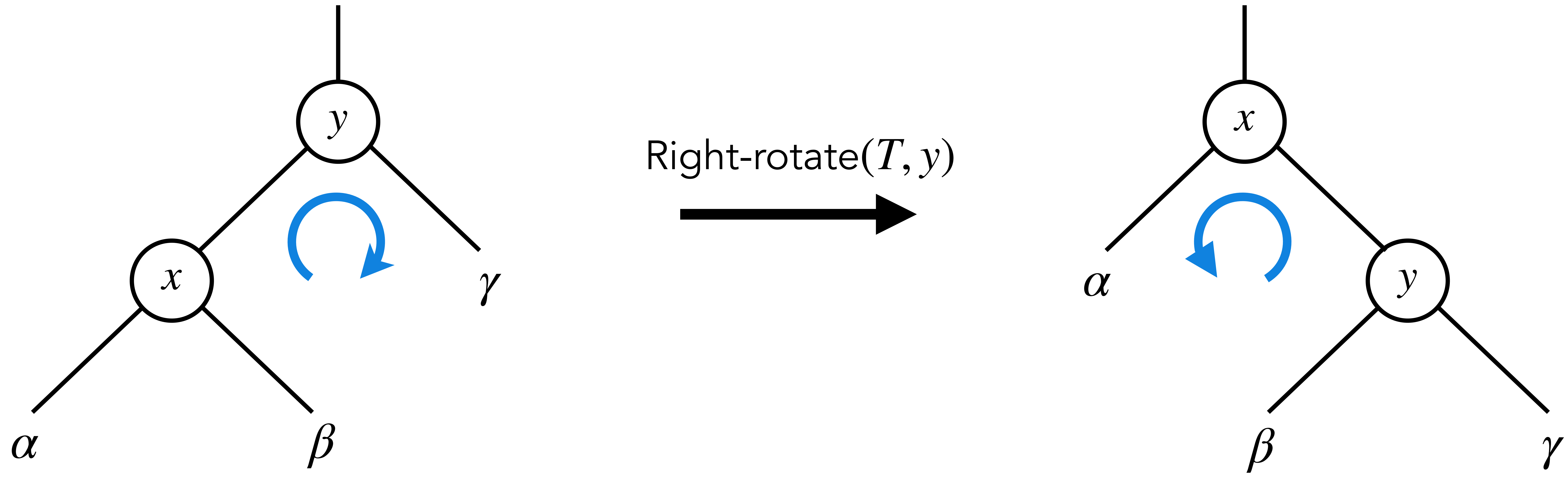

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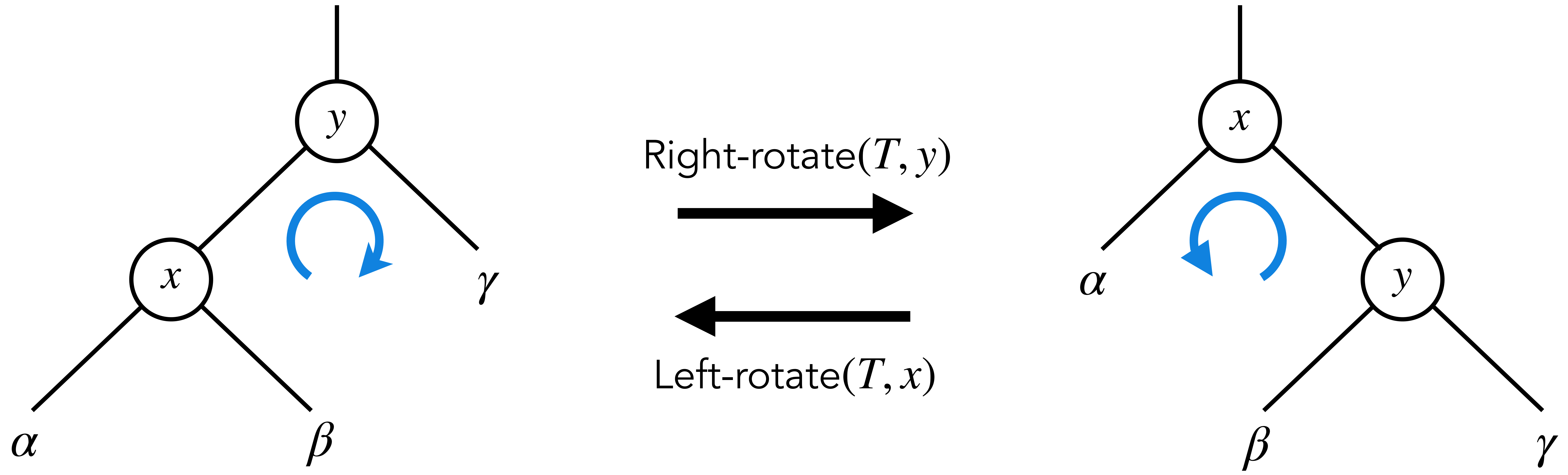

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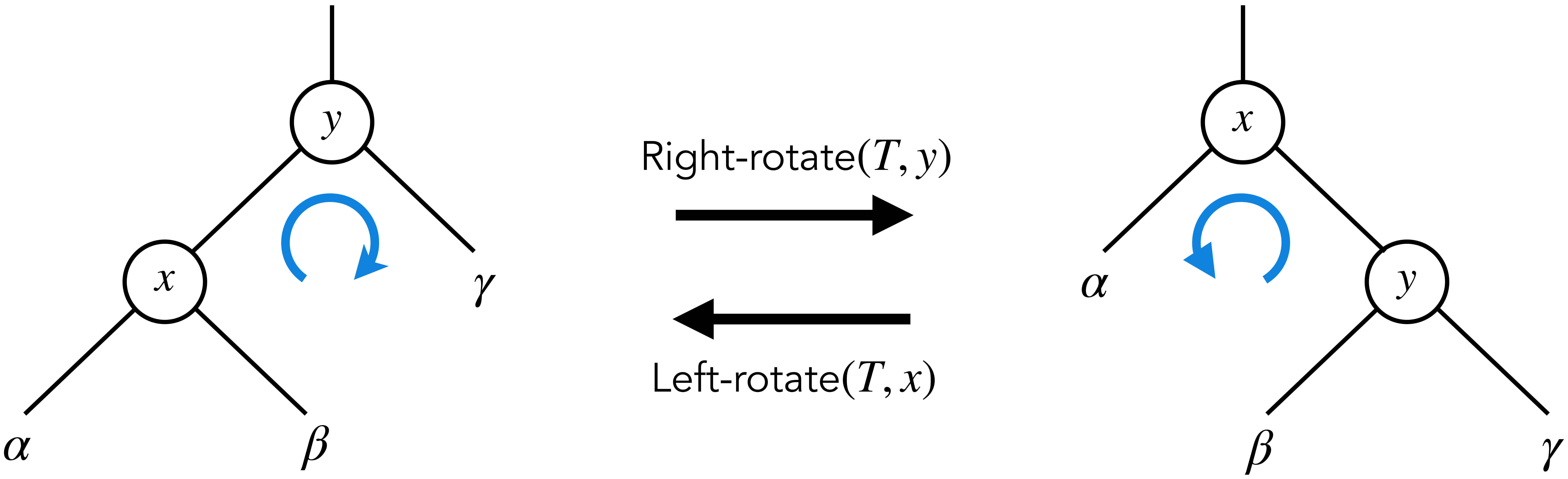

Note: Rotations do not disturb BST property and can be performed in constant time.

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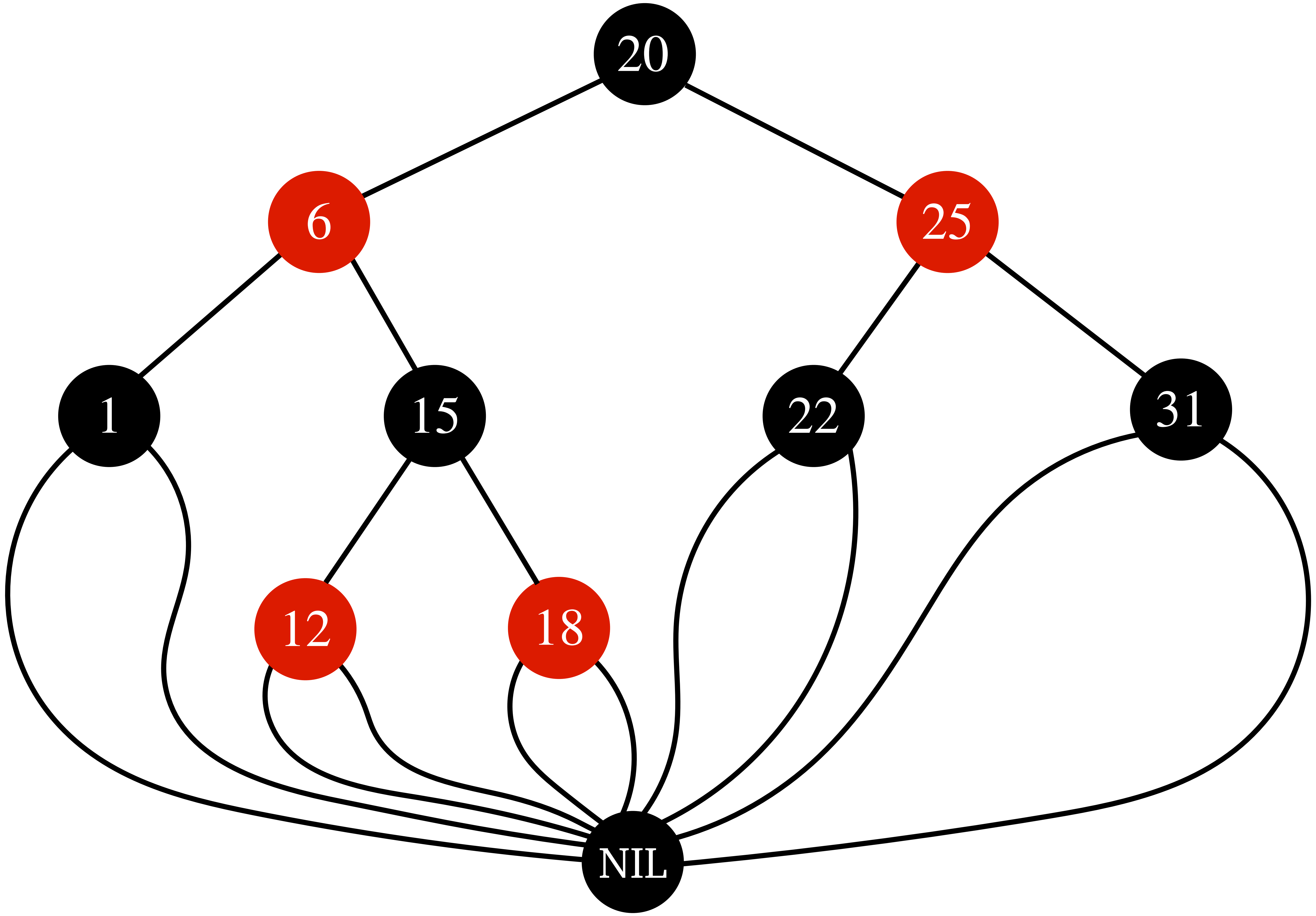

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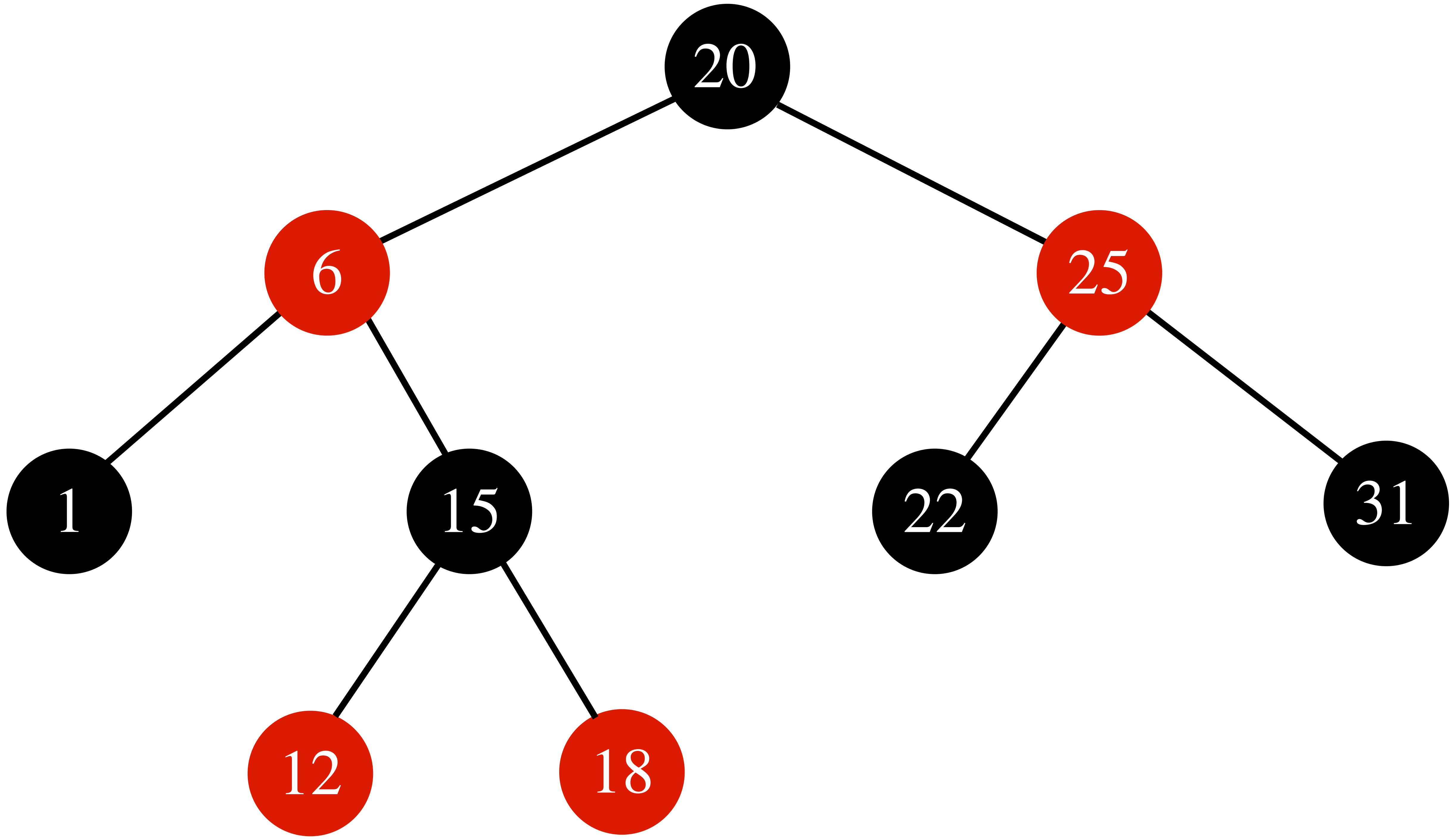

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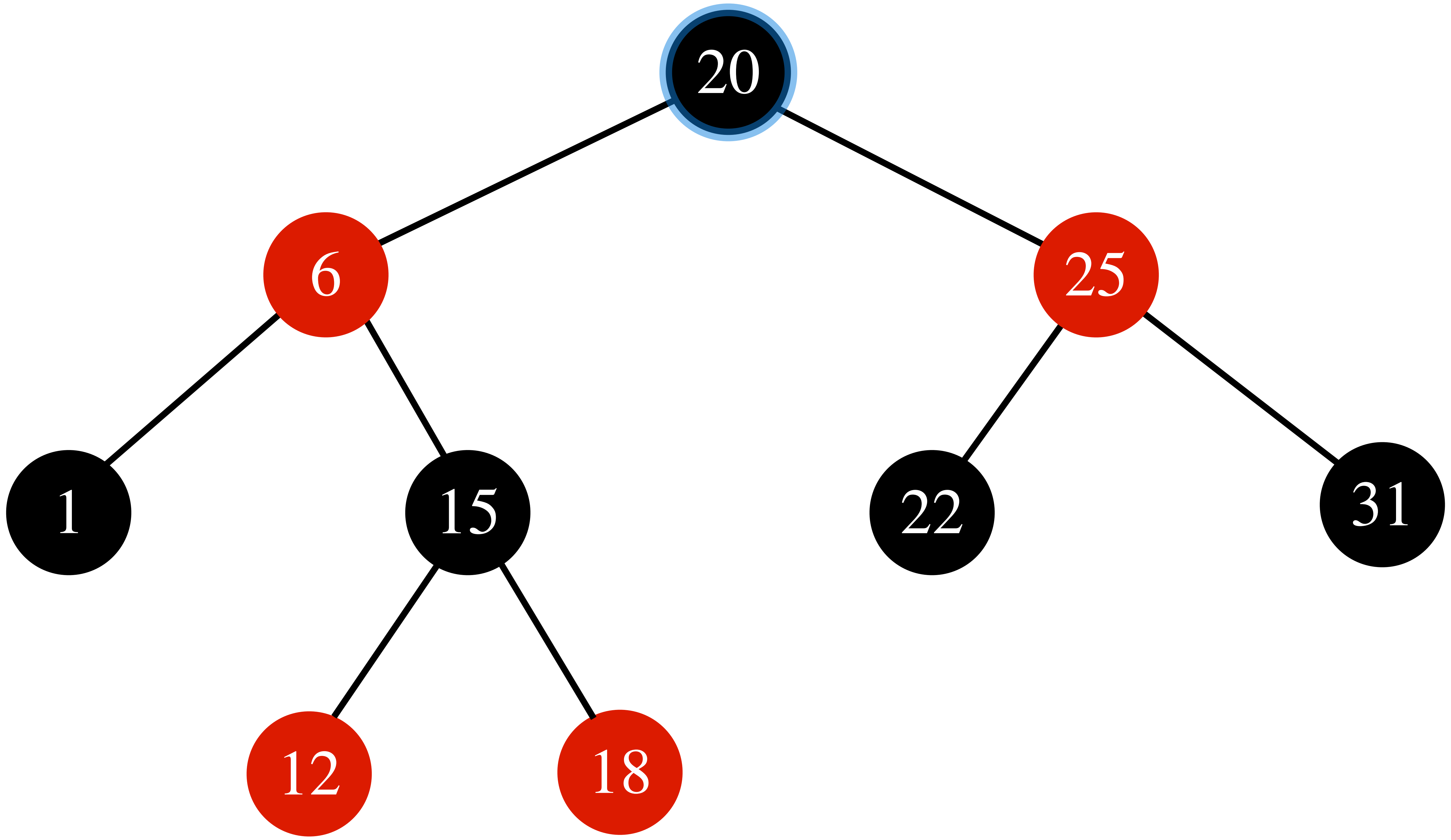

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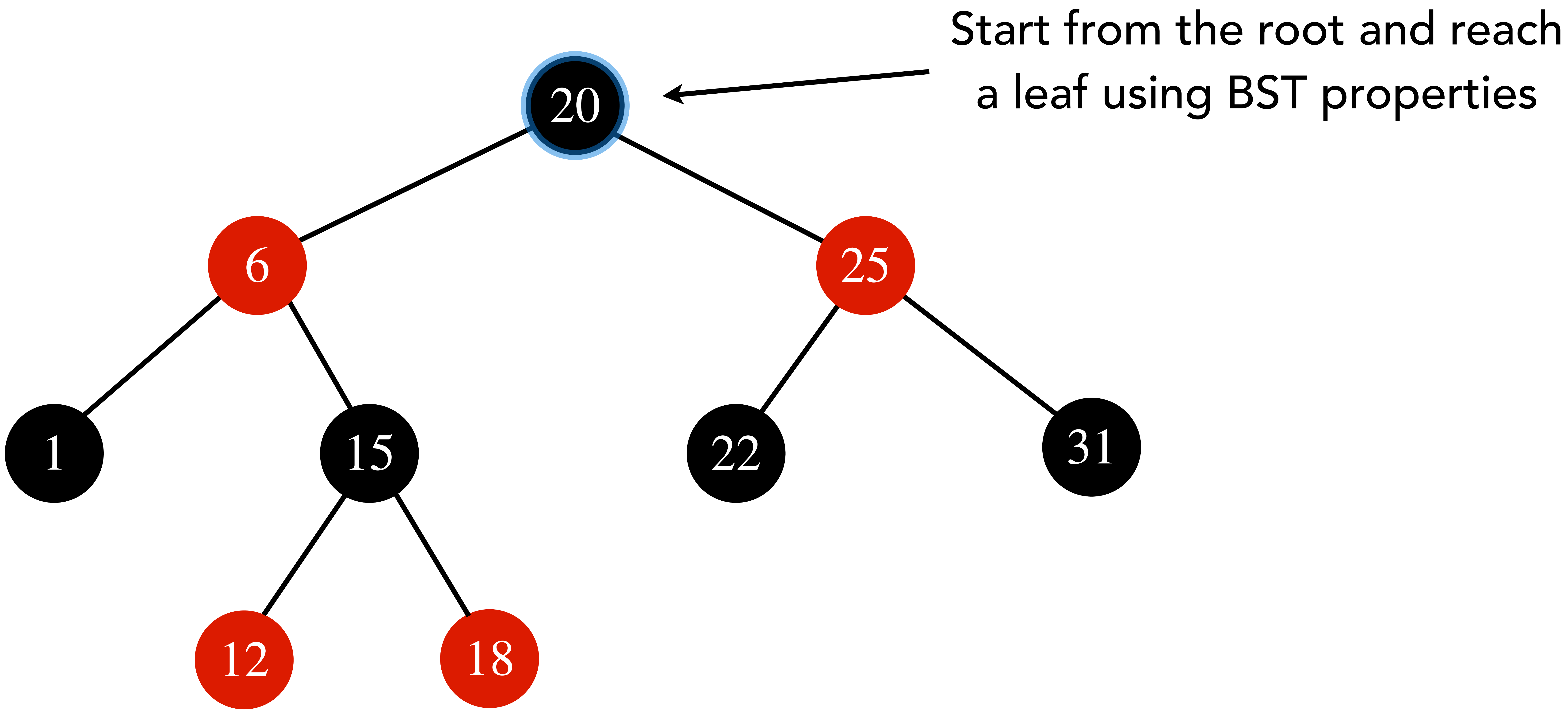

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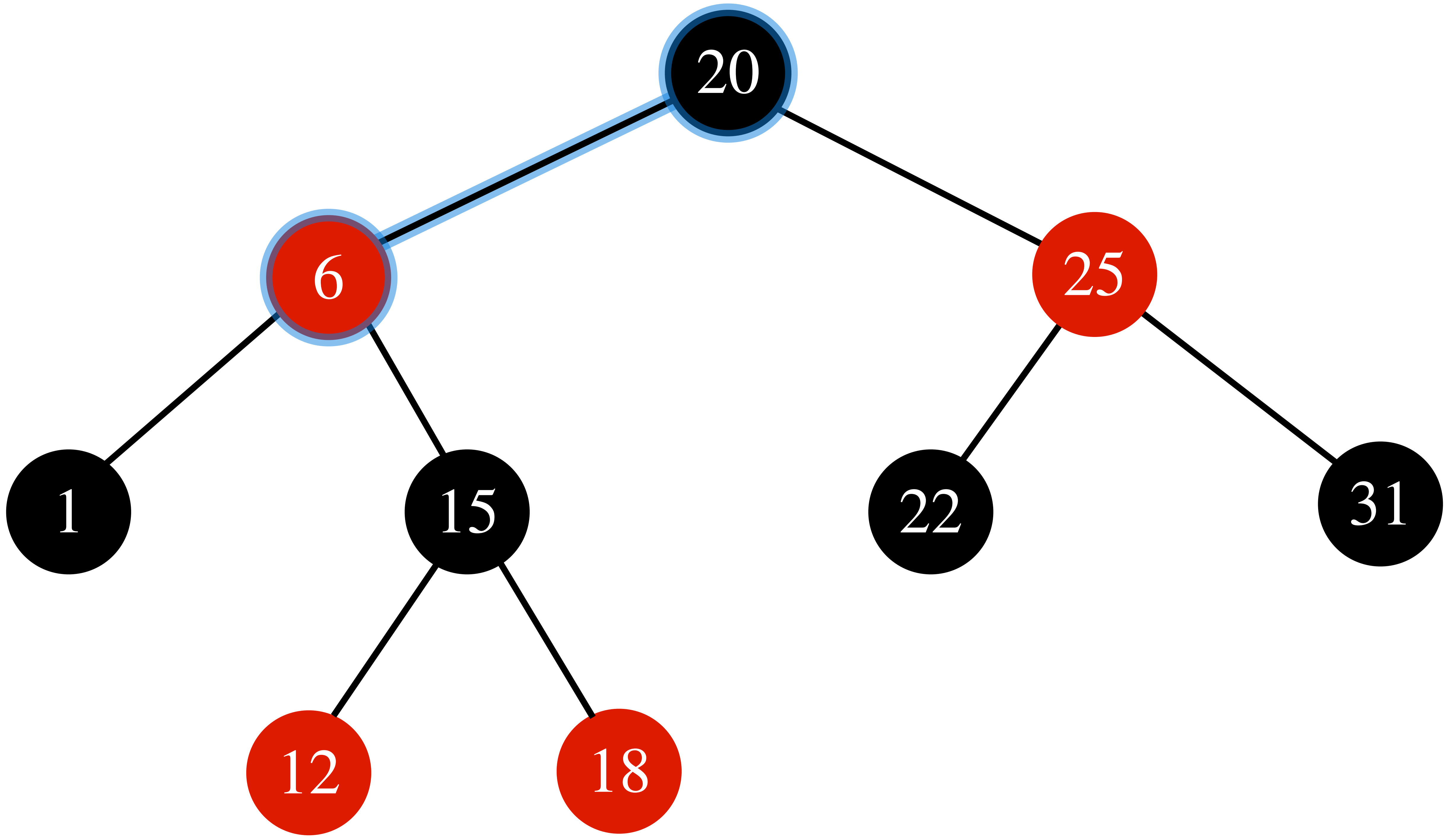

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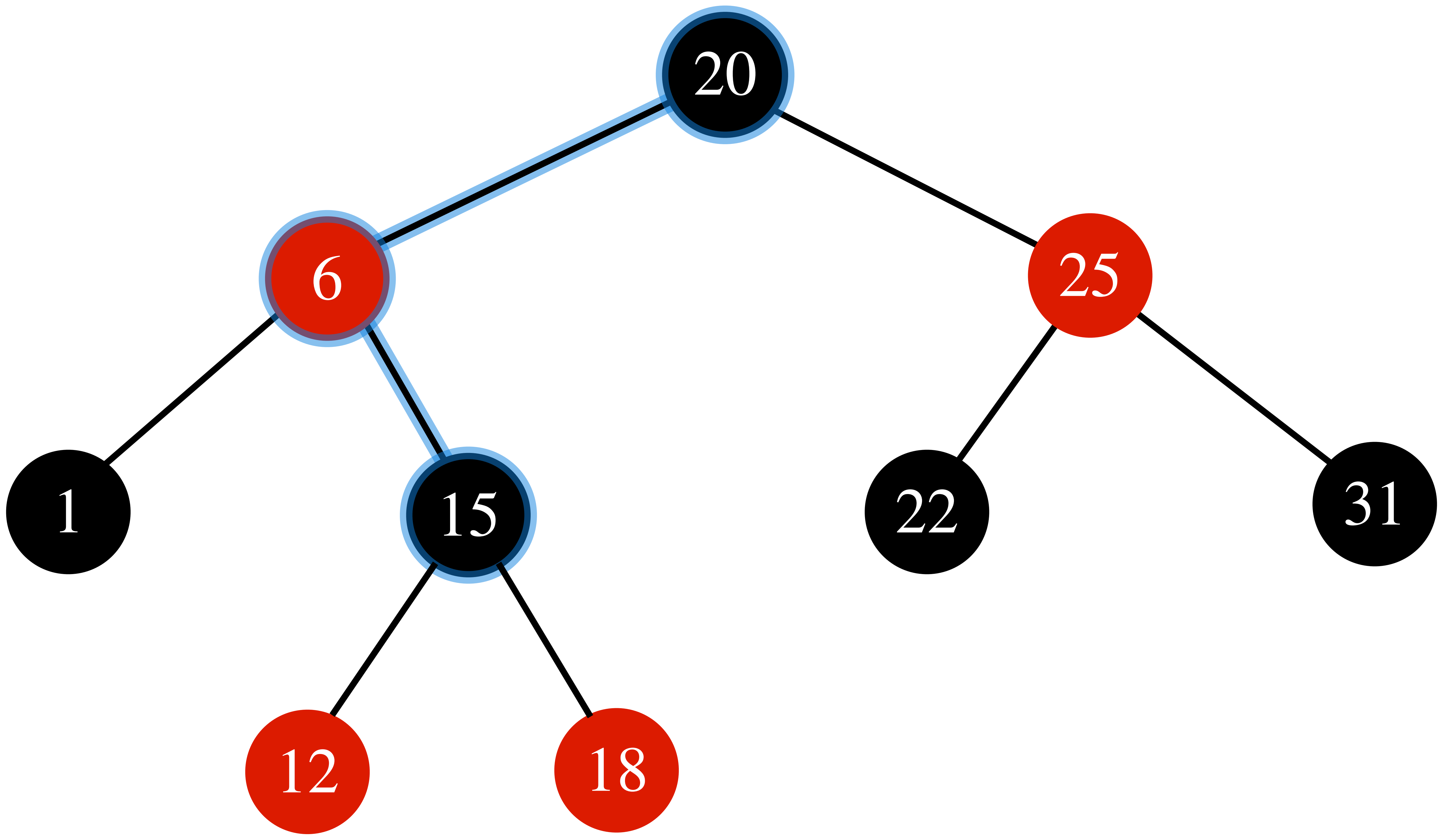

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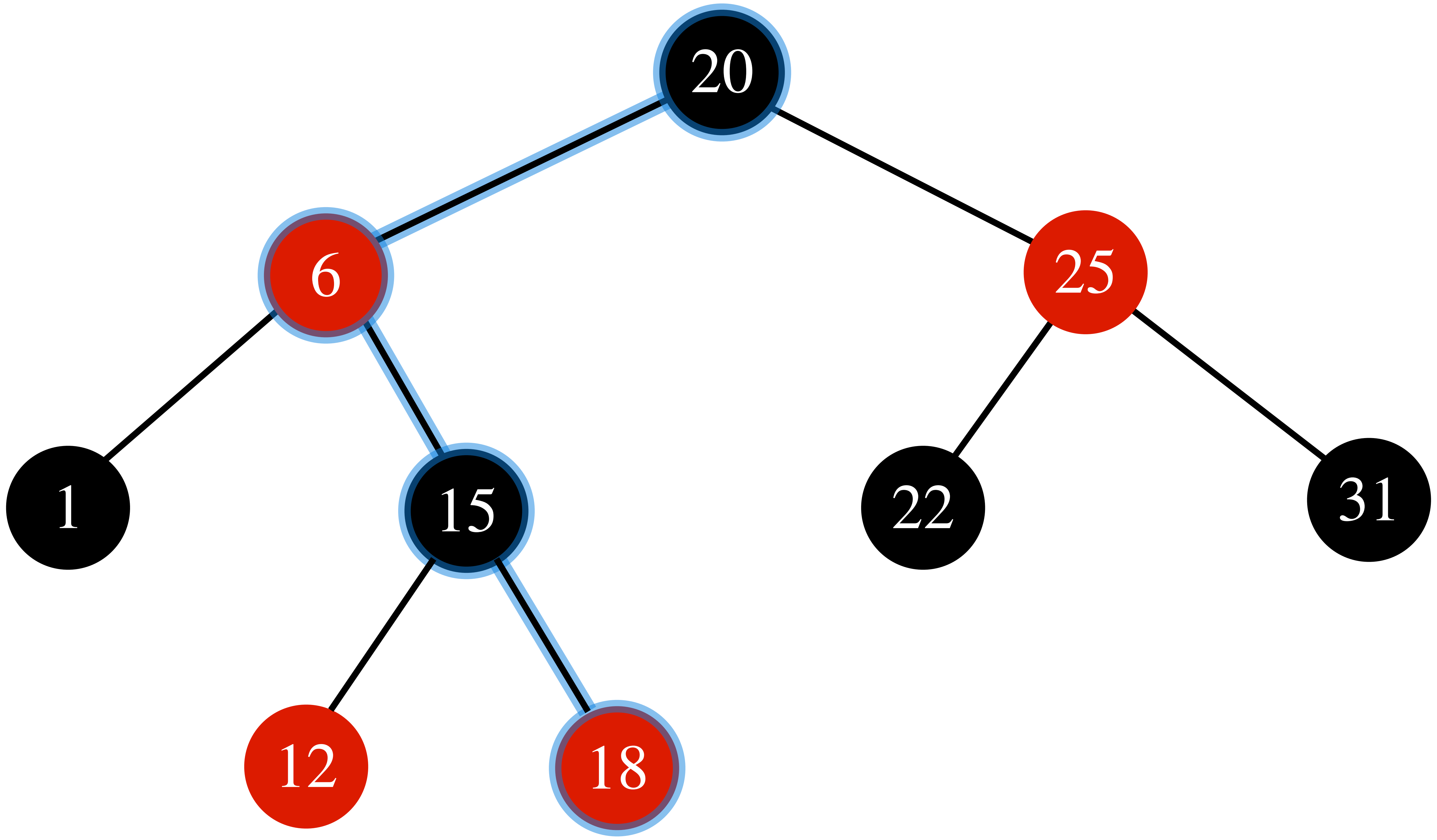

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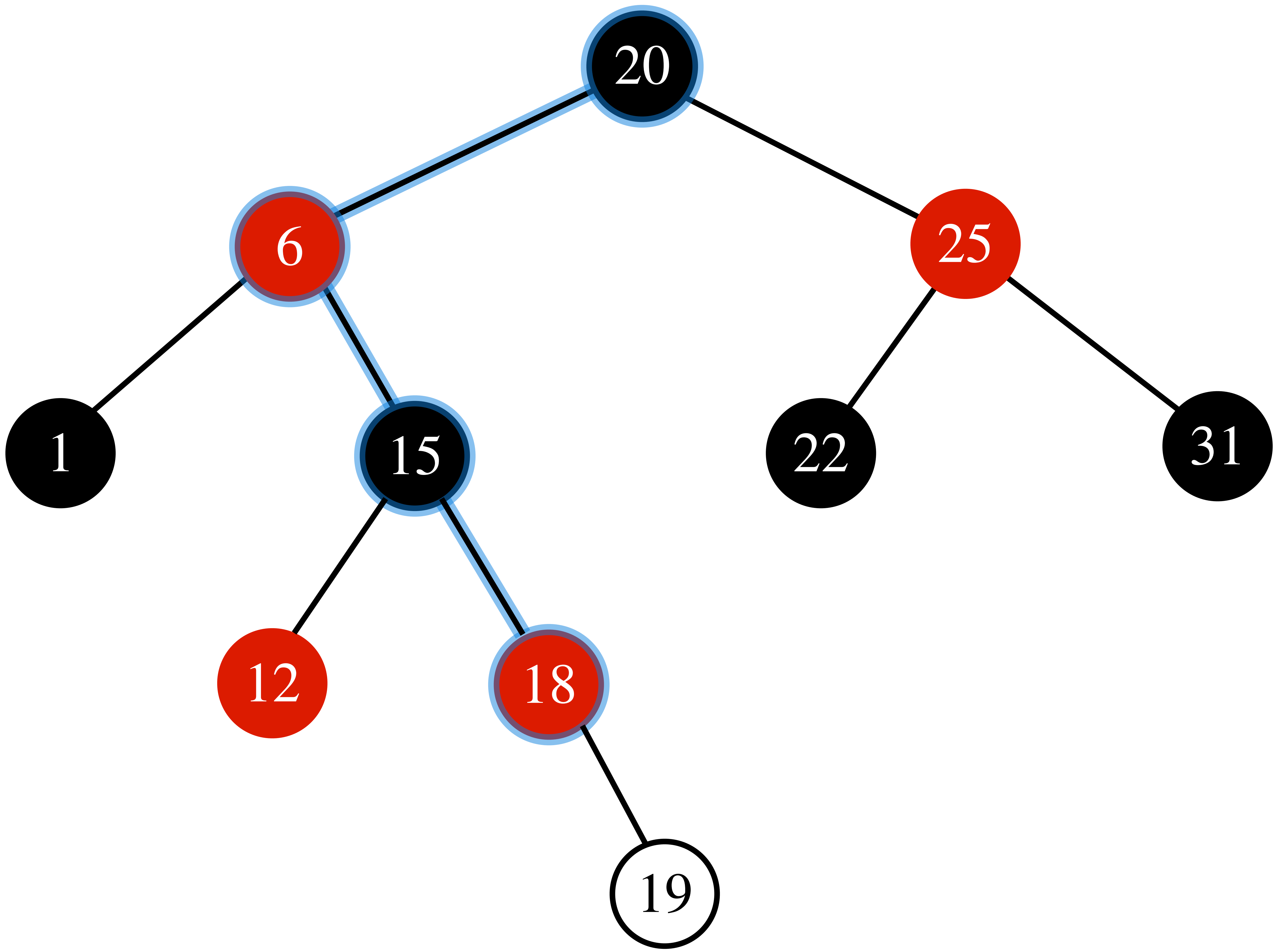

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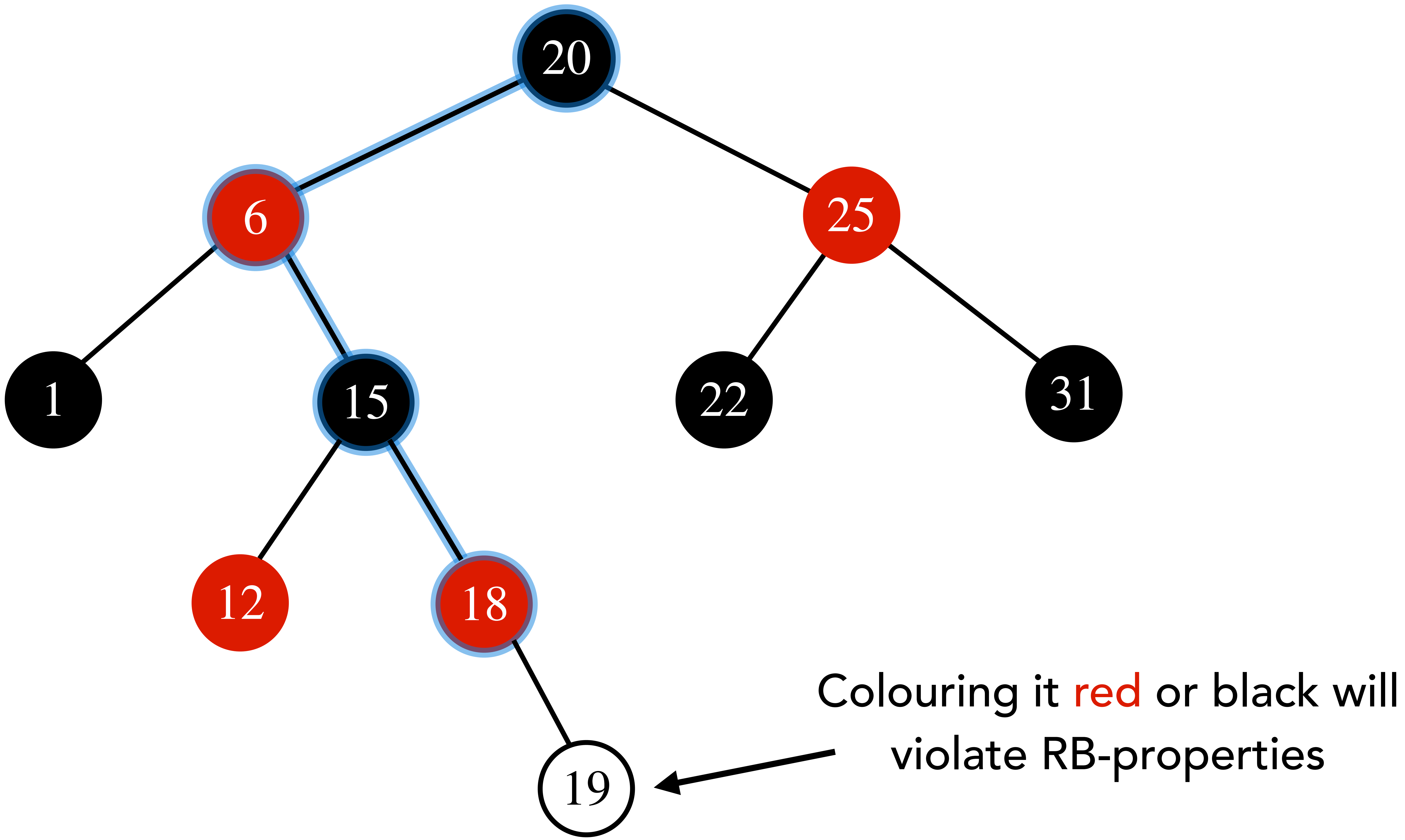

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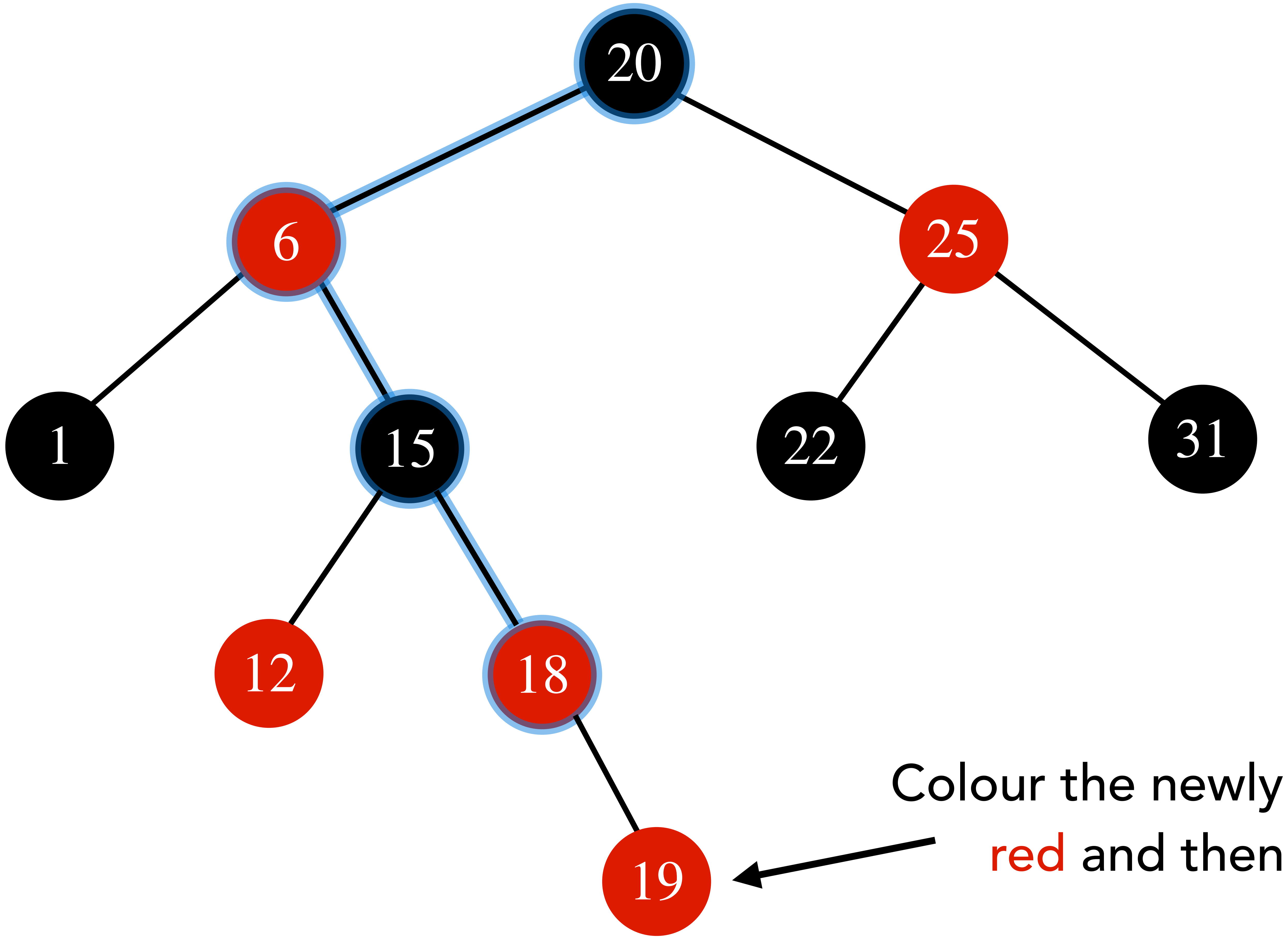

Colour the newly inserted node red and then do fix-up.

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- Insert the new node as it is done in a BST and colour the new node red.
- Do fix-ups as parent of the new node may also be a red node.